**Math 675, Fall 2012, Homework 1**

**Due: Thursday, Sept. 6**

1. Textbook, Section 5, problem 1 (pg. 45)

2. Textbook, Section 5, problem 3 (pg. 45)

3. Textbook, Section 5, problem 6 (pg. 45)

4. Textbook, Section 5, problem 7 (pg. 45)

5. Stereographic projection creates a strange metric for the usual 2-D plane of points $(x, y)$ by considering it in 3-D as points of the form $(x, y, 0)$ and creating a 1:1 mapping to points other than $(0, 0, 1)$ **on the unit sphere** in 3-D. You do this by finding the unique other point on the unit sphere in 3-D which lies on the line segment connecting $(x, y, 0)$ to $(0, 0, 1)$. Write out this function. Then use the usual Euclidean distance between points in 3-D to find a strange distance formula for the plane by defining the strange distance between points in the plane as the Euclidean distance between their images on the sphere. Explain why is this a metric for the plane. Since the point $(0, 0, 1)$ has no image in the plane, we say it then corresponds to "an ideal point at $\infty$" and we have what is known as a one-point compactification of the original plane. Find as well the distance formula between $(x, y)$ and the point at $\infty$ also.

6. Textbook, Section 6, problem 2 (pp. 53-54)

7. Textbook, Section 6, problem 3 (pp. 53-54)