MATH 413 – HOMEWORK 6 SOLUTION

Homework from 3.1

#1 $f(x) = 1-x \rightarrow w = c\frac{du}{dx} = \int_x^1 f(t)\,dt = +\frac{1}{2}(1-x)^2$

so $c\frac{du}{dx} = +\frac{1}{2}(1-x)^2 \Rightarrow u(x) = \int_0^x +\frac{1}{2c}(1-\cancel{t})^2\,dt = B - \frac{1}{6c}(1-x)^3$

Pick B so $u(0)=0 \Rightarrow u(x) = \frac{1}{6c}\left[1-(1-x)^3\right]$

#2 $w = \int_x^1 f(t)\,dt = (1-x)\underset{\uparrow\text{constant}}{f}$ and $(1-\cancel{x})\frac{du}{dx} = (1-x)f$ so $u(x) = xf$

#5 $-u'' = e^x \Rightarrow u(x) = -e^x + A + Bx$ with $u(0) = u(1) = 0$

$-1 + A = 0 \rightarrow A = 1 \rightarrow -e + 1 + B = 0,\ B = e-1$

Answer $u(x) = -e^x + 1 + (e-1)x$

#6 $-\frac{dw}{dx} = f \Rightarrow w = (1-x)f \Rightarrow c\frac{du}{dx} = (1-x)f$

$\frac{du}{dx} = (1-x)f$ for $x \le 1/2 \Rightarrow \boxed{u(x) = \left(x - \frac{x^2}{2}\right)f,\ x \le 1/2}$

now: $u(1/2) = \left(\frac{1}{2} - \frac{1}{8}\right)f = 3/8 f \Rightarrow u'(x) = \frac{1}{2}(1-x)f$ $x > 1/2$ with $u(1/2) = 3/8 f$

$u(x) = \frac{3}{8}f + \int_{1/2}^x \frac{1}{2}(1-t)f\,dt = \frac{3}{8}f + \left(\frac{1}{2}t - \frac{1}{4}t^2\right)f\Big|_{1/2}^x$

$= \frac{1}{2}xf - \frac{1}{4}x^2 f + \left(3/8 - 1/4 + \frac{1}{16}\right)f = \boxed{u(x) = \frac{1}{2}xf - 1/4 x^2 f + \frac{3}{16}f \quad x > 1/2}$

#15 $-u'' = f = \delta_{1/2}(x) \Rightarrow u = \begin{cases} Ax + B & \text{for } x < 1/2 \\ Cx + D & \text{for } x > 1/2 \end{cases}$

$u'(1) = C = 0 \Rightarrow u = D,\ x > 1/2$

$u = Ax,\ x < 1/2$ for $u(0) = 0 \Rightarrow u' = \overset{\text{jump}\,0}{A} \Rightarrow A = 1$

$u(1/2)$ match $\Rightarrow u = \begin{cases} x \\ 1/2, & x \ge 1/2 \end{cases}$

#16 $\begin{cases} Ax+B & x<1/2 \\ Cx+D & x>1/2 \end{cases}$ but $u(0)=u(1)$ and $u' = \begin{cases} A & x<1/2 \\ C & x>1/2 \end{cases}$

get $\left.\begin{array}{ll} Ax, & x<1/2 \\ -C(1-x), & x>1 \end{array}\right]$ match at $1/2$: $A \cdot 1/2 = -C \cdot 1/2$

jump in u': A to $+C = -A$ so $A = 1/2$