MATH 413 — HMWK 3 — SOLN

1.6.2 — by inspection / def$^{\underline{n}}$

$$A_1 = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$A_1 x = b$ has sol$^{\underline{n}}$ if $b_1 + b_2 + b_3 + b_4 = 0$ ie. $[1\ 1\ 1\ 1]\, b = 0$. The vectors $c[1\ 1\ 1\ 1]^T$ are in null space of A_1^T.

1.6.34 $B = I - vw^T$, $B^{-1} = I - c\,v\,w^T$ - find c

$$BB^{-1} = I = (I - vw^T)(I - cvw^T) = I - vw^T - cvw^T + cvw^Tvw^T$$

now w^Tv is scalar so we get I $\underline{\text{if}}$

$$cw^Tv - c - 1 = 0 \quad \text{so} \quad c = \frac{1}{w^Tv - 1}$$

provided $w^Tv \neq 1$. If $w^Tv = 1$, then B is not invertible since $Bv = Iv - vw^Tv = v - v\cdot(w^Tv) = 0$ then.

1.6.35 Similar to 34 $\rightarrow$

$$(A - vw^T)(A^{-1} - cA^{-1}vw^TA^{-1})$$

$$= AA^{-1} - vw^TA^{-1} - cAA^{-1}vw^TA^{-1} + vw^TcA^{-1}vw^TA^{-1}$$

$$= I - vw^TA^{-1} - cvw^TA^{-1} + cv\underbrace{[w^TA^{-1}v]}_{\text{scalar}}w^TA^{-1}$$

$= I$ if the coeff. of vw^TA^{-1} is 0 — this says

$$-1 - c + cw^TA^{-1}v = 0 \quad \text{or} \quad c(w^TA^{-1}v - 1) = 1$$

so $c = \dfrac{1}{w^TA^{-1}v - 1}$ if $w^TA^{-1}v - 1 \neq 0$.

Note $B = A - vw^T$ so $B(A^{-1}v) = A(A^{-1}v) - vw^TA^{-1}v$

$$= v - v[w^TA^{-1}v] = 0$$

if $w^TA^{-1}v = 1$

(b) with $v = w = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ then B is A with -1 in $(1,1)$ entry

Formula says $B^{-1} = A^{-1} - cA^{-1}vw^TA^{-1}$

with $c = [w^TA^{-1}v - 1]^{-1} = (s-1)^{-1}$ in this case

and $A^{-1}v = q$ — 1st col. of A^{-1}

$w^TA^{-1} = r^T$ — 1st row

get $(s-1)^{-1}qr^T$ subtracted from A^{-1}