MATH 413 Homework 2 – SOLN

1.4.1 – (i) $-\frac{1}{2}(x_1^2+x_2^2) \leftarrow x_1 b_1 - x_2 b_2$ Value $-\frac{1}{2}b_1^2 - \frac{1}{2}b_2^2$

$A = I \quad A\vec{x} - \vec{b} = \vec{0} \Rightarrow x_1 = b_1$

$x_2 = b_2$

(ii) $P = \frac{1}{2}(x_1^2 + 2x_1x_2 + 3x_2^2) - x_1 + x_2$

$P_{x_1} = x_1 + x_2 - 1 = 0$

$P_{x_2} = x_1 + 2x_2 + 1 = 0$

$x_2 = -2$

$x_1 = 3$

Value $\frac{1}{2}(9 - 12 + 8) - 3 - 2$

$= \frac{1}{2}[5] - 5 = \boxed{-5/2}$ ✓

1.4.2 – (i) $A\vec{x} - \vec{b} = \vec{0}$

(ii) $A^TA\vec{x} - A^Tb = \vec{0}$

(iii) $A^TA\vec{x} - A^Tb = \vec{0}$

1.4.7 – (i) $y = C \rightarrow C = 0$, $C = 3$, $C = 12$ $\Leftrightarrow \begin{pmatrix}1\\1\\1\end{pmatrix} C = \begin{pmatrix}0\\3\\12\end{pmatrix}$

$\Leftrightarrow \begin{pmatrix}1&1&1\end{pmatrix}\begin{pmatrix}1\\1\\1\end{pmatrix} C = \begin{pmatrix}1&1&1\end{pmatrix}\begin{pmatrix}0\\3\\12\end{pmatrix} = 15$

$3C = 15, \quad \boxed{C = 5} \rightarrow 5^2 + 2^2 + 7^2$

Sum of squares

(ii) $C + 0D = 0$

$C + D = 3$

$C + 2D = 12$

$\begin{pmatrix}1&0\\1&1\\1&2\end{pmatrix}\begin{pmatrix}C\\D\end{pmatrix} = \begin{pmatrix}1&1&1\\0&1&2\end{pmatrix}\begin{pmatrix}0\\3\\12\end{pmatrix}$

$\begin{pmatrix}3&3\\3&5\end{pmatrix}\begin{pmatrix}C\\D\end{pmatrix} = \begin{pmatrix}15\\27\end{pmatrix}$

$3C+3D=15$
$3C+5D=27$ } $2D=12$

$D=6$
$C=-1$

$-1 \leftrightarrow 0$ $\qquad 1^2+2^2+1^2$

$5 \leftrightarrow 3$

$11 \leftrightarrow 12$

$C+0D+0E=0$
$C+D+E=3$ 3 eqns, 3 unknowns
$C+2D+4E=12$

$C=0$
$D=0$
$E=3$

not $3t^2 \rightarrow$ exact.

1.48

t	z	b
0	0	2
0	1	2
1	0	1
1	1	5

$C+Dt+Ez=b$

$C=2$
$C+E=2$
$C+D=1$
$C+D+E=5$

$$\begin{pmatrix}1&1&1&1\\0&0&1&1\\0&1&0&1\end{pmatrix}\begin{pmatrix}1&0&0\\1&0&1\\1&1&0\\1&1&1\end{pmatrix}\begin{pmatrix}C\\D\\E\end{pmatrix}=\begin{pmatrix}1&1&1\\0&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}2\\2\\1\\5\end{pmatrix}=\begin{pmatrix}10\\6\\7\end{pmatrix}$$

$$\begin{pmatrix}4&2&2\\2&2&1\\2&1&2\end{pmatrix}\begin{pmatrix}C\\D\\E\end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 4 & 2 & 2 & 10 \\ 2 & 2 & 1 & 6 \\ 2 & 1 & 2 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 2 & 2 & 1 & 6 \\ 2 & 1 & 2 & 7 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 2 & 1 & 1 & 5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right) \Leftrightarrow \begin{array}{l} C=1 \\ D=1 \\ E=2 \end{array}$$

Model $\boxed{1+t+2z}$ → predicted

		actual
1	2	1^2
3	2	1^2
2	1	1^2
4	5	1^2