HOMEWORK SOLUTIONS MATH 116 Problems in 9.1 / 7.4

9.1 #9 (a) $f(x) = (1+x)^{-1}$ - Linear at $a=0$ $f(0)+f'(0)\cdot x$

$f(0)=1$, $f'(x) = (-1)(1+x)^{-2}$ so $f'(0) = -1 \rightarrow$ Linear approx $\boxed{1-x}$

(b) $f''(x) = (-1)(-2)(1+x)^{-3}$ so $f''(0) = 2$ - Quadratic is $\boxed{1-x+x^2}$

(c) For $\frac{1}{1.05} = f(0.05)$ we have: Linear approx $1-0.05 = 0.95$

Quadratic $1-0.05+(0.05)^2 = 0.9525$

answer is ≈ 0.95238095 - quadratic is better than linear, both pretty good.

#21 $(1+x)^{1/2}$: Taylor, order 2 yields $1 + x/2 - x^2/8 = p_2(x)$

$f(0.05) = (1.05)^{1/2} \approx 1.02469508$ while $p_2(0.05) = 1 + 0.025 - 0.0003125$

$= 1.0246875$

Error $= 0.00000757$

#67 (a) $(1+2x)^{1/2}$: $f(0)=1$, $f'(x) = \frac{1}{2}(1+2x)^{-1/2}\cdot 2$ so $f'(0) = 1$ also

To get these, find derivatives

also $f''(x) = -\frac{1}{2}(1+2x)^{-3/2}\cdot 2$ so $f''(0) = -1$

(b) $(1+2x)^{-1/2}$: $f(0)=1$, $f'(x) = -\frac{1}{2}(1+2x)^{-3/2}\cdot 2$ so $f'(0) = -1$

$f''(x) = 3/2\,(1+2x)^{-5/2}\cdot 2$ so $f''(0) = 3$ (see (a)) but derivative shifted

(c) e^{2x} : $f(0)=1$, $f'(x) = 2e^{2x}$, $f'(0)=2$, $f''(x) = 4e^{2x}$, $f''(0)=4$

(d) $(1+2x)^{-1}$: $f(0)=1$, $f'(x) = (-1)(1+2x)^{-2}\cdot 2$ so $-2 = f'(0)$

$f''(x) = (-2)(-2)(1+2x)^{-3}\cdot 2$ so $f''(0) = 8$

(e) $(1+2x)^{-3}$: $f(0)=1$, $f'(x) = (-3)(1+2x)^{-4}\cdot 2$ so $f'(0) = -6$

$f''(x) = (-6)(-4)(1+2x)^{-5}\cdot 2$ so $f''(0) = 48$

(f) e^{-2x}: $f(0)=1$, $f'(x) = -2e^{-2x}$: $f'(0) = -2$

$f''(x) = 4e^{-2x}$, $f''(0) = 4$

Taylor polys. are $f(0)+f'(0)x + \frac{1}{2}f''(0)x^2$ so matching is easy now – only need f'' to pick for **D** vs. F.

Thus (a) – C, (b) – E, (c) A, (d) D (e) B (f) F

$\boxed{7.4}$ Problem 9 $\displaystyle\int \frac{dx}{(x-1)(x+2)} = \int\left(\frac{A}{x-1} + \frac{B}{x+2}\right)dx$

$= A\ln|x-1| + B\ln|x+2| + C$ – find A, B: $\dfrac{A}{x-1} + \dfrac{B}{x+2} = \dfrac{1}{(x-1)(x+2)}$

so $A = \lim\limits_{x\to 1}\dfrac{1}{(x+2)} = \dfrac{1}{3}$ ("cover up"), $B = \lim\limits_{x\to -2}\dfrac{1}{x-1} = -\dfrac{1}{3}$

or $A(x+2) + B(x-1) = 1$ – common denom

use $x = 1, -2$

or $A(x+2) + B(x-1) = 1$

so $x(A+B) + (2A-B) = 0\cdot x + 1$

$A+B = 0$, $2A-B = 1$ > add $3A = 1$, $A = \frac{1}{3}$

then $B = -A = -\frac{1}{3}$

$$\boxed{\text{ans } \tfrac{1}{3}\ln|x-1| - \tfrac{1}{3}\ln|x+2| + C}$$

\#19 $\displaystyle\int \frac{3}{x^3-9x^2}\,dx$ split: $\dfrac{A}{x^2} + \dfrac{B}{x} + \dfrac{C}{x-9} = \dfrac{3}{x^3-9x^2}$

Common denom yields: $A(x-9) + Bx(x-9) + Cx^2 = 3$

Now use limits at $x=9$, $x=0$ to get

$C\cdot 9^2 = 3$ so $C = 3/9^2 = 1/27$, $-9A = 3$ so $A = -1/3$

We have $-\frac{1}{3}(x-9) + Bx(x-9) + \frac{1}{27}x^2 = 3$: From x^2 term get $B = -\frac{1}{27}$

(or take derivative, then set $x=0$)

so $\displaystyle\int \frac{3}{x^3-9x}\,dx = \int\left[-\tfrac{1}{3}\cdot x^{-2} + \left(-\tfrac{1}{27}\right)x^{-1} + \tfrac{1}{27}(x-9)^{-1}\right]dx$

$$= \boxed{\frac{1}{3}x^{-1} - \frac{1}{27}\ln|x| + \frac{1}{27}\ln|x-9| + C}$$

#30 $\displaystyle\int \frac{x^2+2}{x(x^2+5x+8)}\,dx = \int\left[\frac{A}{x} + \frac{Bx+C}{x^2+5x+8}\right]dx$

algebra: $A(x^2+5x+8) + x\cdot(Bx+C) = x^2+2$

Limit, $x\to 0$: $8\cdot A = 2$, $\boxed{A = 1/4}$ so $\frac{1}{4}x^2 + \frac{5}{4}x + 2 + x(Bx+C) = x^2+2$

or, $\frac{1}{4}x^2 + \frac{5}{4}x + x(Bx+C) = x^2$ so $Bx+C = \frac{3}{4}x - \frac{5}{4}$ after division by x.

$\int \frac{1}{4}\cdot x^{-1}\,dx = \frac{1}{4}\ln|x|$, but $\displaystyle\int \frac{\frac{3}{4}x - \frac{5}{4}}{x^2+5x+8}\,dx$ needs some work.

By "completing the square" we find we should let $u = x+5/2$ then

$u^2 = (x+5/2)^2 = x^2 + 2\cdot\frac{5}{2}\cdot x + (5/2)^2 = x^2+5x+\frac{25}{4}$ so $x^2+5x+8 = u^2 + 7/4$

our integral becomes $\displaystyle\int \frac{\frac{3}{4}(u - 5/2) - \frac{5}{4}}{u^2 + 7/4}\,du$

$$= \frac{3}{4}\int \frac{u}{u^2+7/4}\,du - \frac{25}{8}\int \frac{1\,du}{u^2+7/4} = \frac{3}{8}\ln\left(u^2+7/4\right)$$

$$- \frac{25}{8}\left(\frac{1}{\sqrt{7}/2}\right)\tan^{-1}\left(\frac{u}{\sqrt{7}/2}\right) + C$$

so we get, going back to x:

$$\boxed{\frac{1}{4}\ln|x| + \frac{1}{8}\ln\left(x^2+5x+8\right) - \frac{25}{4\sqrt{7}}\tan^{-1}\left(\frac{x+5/2}{\sqrt{7}/2}\right) + C}$$

SOME ISSUES that came up —

1. Which x values to use in Taylor polynomials
2. Algebra for partial fractions
3. u-substitution for quadratic denominators.

WILL LOOK AT TAYLOR APPLETS IN REVIEW