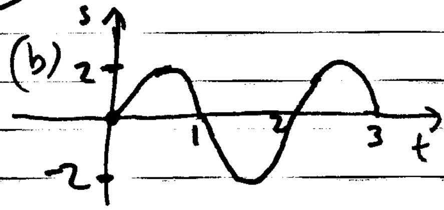
Math 116 – Spring 2011 – Homework 1 Solutions – Prof SACHS

$\boxed{6.1}$ #15 (a) $s(0)$, $s'(t) = v(t) = 2\pi \cos \pi t$ so $s(t) = 2\sin(\pi t)$

(b)



(c) lowest at $\boxed{\tfrac{3}{2}}$, $\tfrac{3}{2}+2 = \boxed{\tfrac{7}{2}}$, $\tfrac{7}{2}+2 = \boxed{\tfrac{11}{2}}$

(d) highest at $\boxed{\tfrac{1}{2}}$, $\tfrac{1}{2}+2 = \boxed{\tfrac{5}{2}}$, $\tfrac{5}{2}+2 = \boxed{\tfrac{9}{2}}$

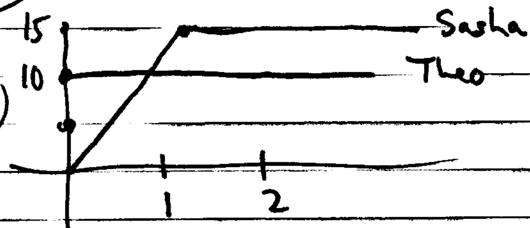
#22 $a(t) = e^{-t}$ so $v(t) = -e^{-t} + C$, $v(0) = -1 + C = 60$ so $C = 61$

$v(t) = -e^{-t} + 61$ so $s(t) = e^{-t} + 61t + K$ ← random letter for constant

$s(0) = 1 + 61 \cdot 0 + K = 40$ so $K = 39$: answers $\boxed{\begin{array}{l} s(t) = 39 + e^{-t} + 61t \\ v(t) = -e^{-t} + 61 \end{array}}$

#47 Bike race $v_T(t) = 10$, $t \geq 0$ while $v_S(t) = 15t$, $0 \leq t \leq 1$ and $v(t) = 15$, $t > 1$

(a)



(b) In one hour, Theo rides 10 miles
Sasha rides 7.5 miles – area of rectangle vs triangle

(c) In two hours, Theo rides 20 miles – rectangle
Sasha rides $7.5 + 15 = 22.5$ miles – trapezoid

(d) From (b), Theo hits 10 miles sooner; Theo takes 1.5 hrs for 15 miles while Sasha takes the same time $(7.5 + \tfrac{1}{2}(15) = 15)$; from there, Sasha moves ahead at 5 mph, so Sasha hits 20 first.

(assume it means 20 mi without head start)

(e) Theo travels 0.2 mi + $10t$ in time t. Time for Sasha to hit 20 miles is $t = 1 + \frac{12.5}{15} = 1 + \frac{5}{6} = \frac{11}{6}$ hrs. Theo has gone $0.2 + \frac{110}{6}$ miles, which is less. Sasha wins.

(f) (somewhat vague instruction / interpretation) start Theo and not clock, so Sasha is as above

In 0.2 hr, Theo goes 2 miles [lot bigger head start] After $11/6$ hrs, Sasha has gone 20 miles, Theo has gone $2 + \frac{110}{6} = \frac{122}{6} > 20$ miles

so Theo wins here.

6.2 #15 area: 2 versions (a) $\int_0^{\pi/4} \sin x\, dx + \int_{\pi/4}^{\pi/2} \cos x\, dx$ (top switches)

$= -\cos x \Big|_0^{\pi/4} + \sin x \Big|_{\pi/4}^{\pi/2} = -\cos \pi/4 + \cos 0 + \sin \pi/2 - \sin \pi/4$

$= -\sqrt{2}/2 + 1 + 1 - \sqrt{2}/2 = \boxed{2 - \sqrt{2}}$

(b) use symmetry – $\cos(\pi/2 - x) = \sin(x)$ so area is $2 \cdot \int_0^{\pi/4} \sin x\, dx = 2\left[\frac{-\sqrt{2}}{2} + 1\right]$

same ✓

#28 $y = x^2 - 4x$ and $y = 2x - 8$ intersect when $x^2 - 4x = 2x - 8$

so $x^2 - 6x + 8 = (x-4)(x-2) = 0 \rightarrow$ intersect when $x = 2$ and $x = 4$

(a) In x: top: $y = 0$, bottom – $0 \le x \le 2 : x^2 - 4x$; $2 \le x \le 4 : 2x - 8$ so $\boxed{\int_0^2 (4x - x^2)\,dx + \int_2^4 (8 - 2x)\,dx}$

(b) In y: single set of right/left & express in y

Right: $y = 2x - 8$ so $\frac{y+8}{2} = x_{Right}$ Left: $y = x^2 - 4x$ so $y + 4 = x^2 - 4x + 4 = (x-2)^2$

$x - 2 = \pm\sqrt{y+4}$ on left, $x < 2$ so choose

so Left: $x_{left} = 2 - \sqrt{y+4}$. y range is from $x = 2$ value ($y = -4$) to $y = 0$

$$\boxed{\int_{-4}^{0} \left[\frac{y+8}{2} - \left(2 - \sqrt{y+4}\right)\right] dy = \boxed{\int_{-4}^{0} \left[\frac{1}{2}y + 2 + \sqrt{y+4}\right] dy}}$$

ok like this

(6.8) (#37) $D(x) = 40\,e^{-x/50}$ shirts sold so $R(x)$ = revenue $= x \cdot 40 e^{-x/50}$

Max when $R'(x) = 0$ – Product rule $R'(x) = 40\left[1 \cdot e^{-x/50} + x\left(\frac{-1}{50}\right)e^{-x/50}\right]$

$R' = 0$ when $x = 50$ so \$50 yield biggest revenue

(#40) $R_T^{(t)}$ is defined as $\dfrac{f(t+T) - f(t)}{f(t)}$ for each time t with given $T > 0$.

When $y(t) = y_0 e^{kt}$ for any k and y_0, then using $y(t)$ as $f(t)$ above, relative growth rate over time interval T at start t is

$\dfrac{y_0 e^{k(t+T)} - y_0 e^{kt}}{y_0 e^{kt}}$. Since $y_0 e^{k(t+T)} = y_0 e^{kt} \cdot e^{kT}$ we get

$R_T = \dfrac{y_0 e^{kt} e^{kT} - y_0 e^{kt}}{y_0 e^{kt}} = e^{kT} - 1$. With $T > 0$, this is not at all dependent on when we measure (time t). Also growth if $R_T > 0$ and decay if $R_T < 0$ [sign of k].