INTEGRATION

associations / ideas

$$\int_a^b f(x)\,dx \longrightarrow$$

AREA

DISTANCE from VELOCITY

ANTIDERIVATIVE

VOLUMES

ADDING UP PIECES TO GET A TOTAL

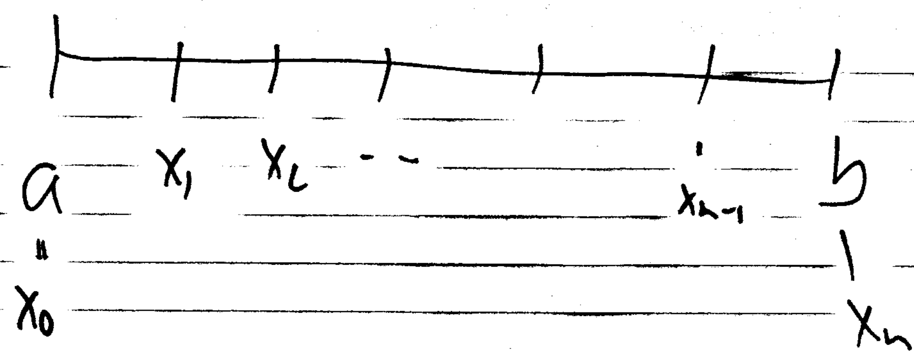
$a < c < b$

$$\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$$

FUNDAMENTAL THEOREM OF CALCULUS

IF $F(x)$ has <u>derivative</u> $f(x)$

then $\int_a^b f(x)\,dx = F(b) - F(a)$



$$F(b) - F(a) = F(x_1) - F(a)$$

START

$$+ \big(F(x_2) - F(x_1)\big)$$

$$+ \dots + F(x_n) - F(x_{n-1})$$

$$= F'(t_1)\,(x_1 - a)$$ — Mean value theorem

$$+ F'(t_2)\,(x_2 - x_1)$$

$$+ \dots + F'(t_n) \cdot (x_n - x_{n-1})$$

= Riemann sum for $\int_a^b F'(x)\,dx$

so $= \int_a^b F'(x)\,dx = \int_a^b f(x)\,dx$

① 6.1 VELOCITY AND NET CHANGE (IN POSITION)

Location $x(t)$

velocity $v(t) = \frac{dx}{dt}(t) = x'(t) = \dot{x}(t)$

acceleration $a(t) = \frac{d^2x}{dt^2}(t) = x''(t) = \ddot{x}(t)$

$$v(t_2) - v(t_1) = \int_{t_1}^{t_2} a(t)\,dt$$

$$x(t_2) - x(t_1) = \int_{t_1}^{t_2} v(t)\,dt$$

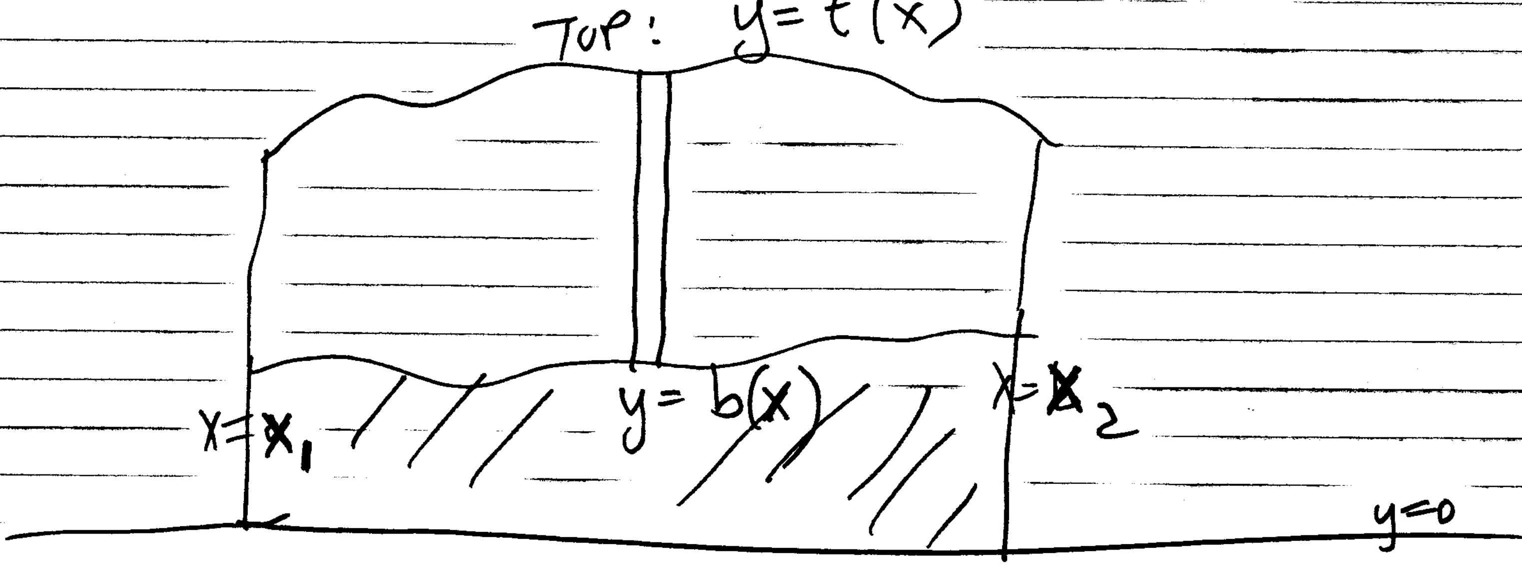
__NOTE__ Distance travelled is __NOT__ $x(t_2) - x(t_1)$

$$s(t_2) - s(t_1) = \int_{t_1}^{t_2} |v(t)|\,dt$$

net ODOMETER reading

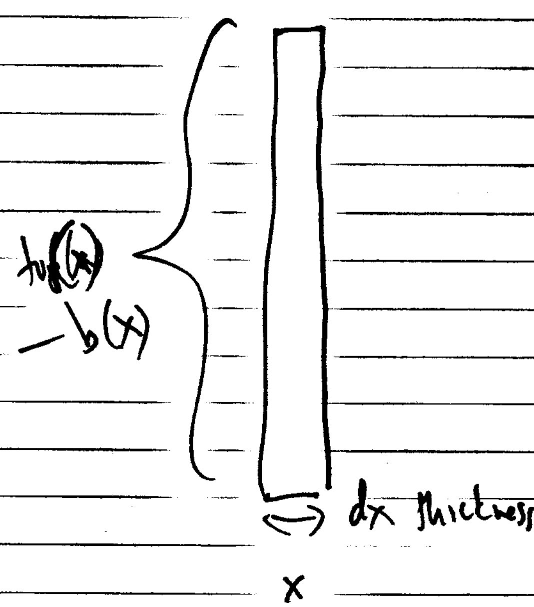
so $\frac{ds}{dt}(t) = |v(t)|$

② 6.2 AREA BETWEEN CURVES



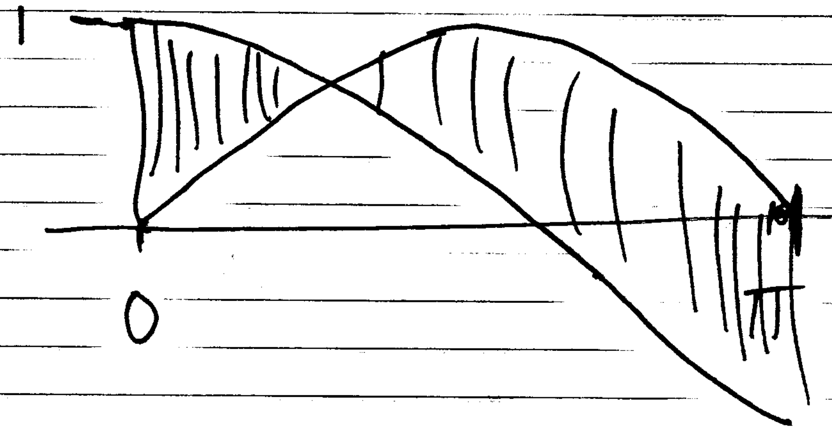
$$\int_{x_1}^{x_2} t(x)\,dx - \int_{x_1}^{x_2} b(x)\,dx$$

$$= \int_{x_1}^{x_2} \left(t(x) - b(x) \right) dx$$



AREA FROM 0 to π

between $\sin x$ and $\cos x$



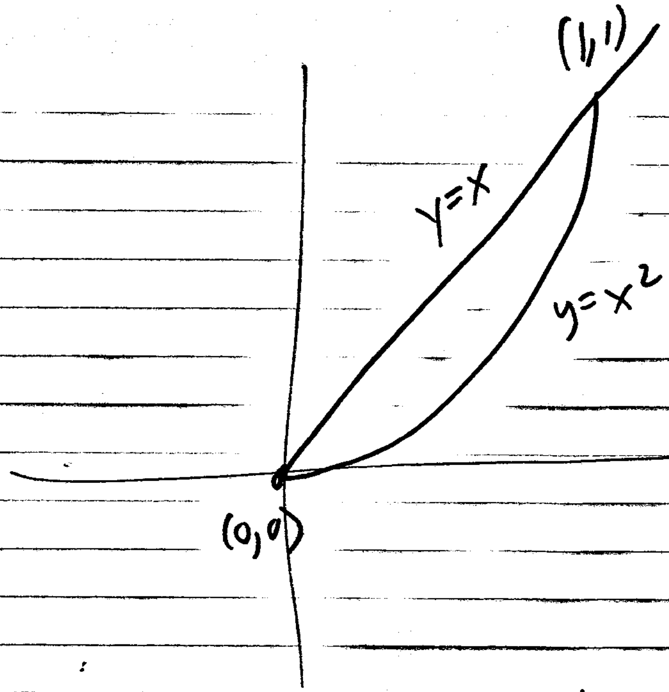
$$\int_0^{\pi} (\cos x - \sin x)\, dx = \sin x + \cos x \Big|_0^{\pi}$$

$$= 0 + \underbrace{\cos(\pi)}_{-1} - \cos(0)$$

$$-1 - 1 = -2$$

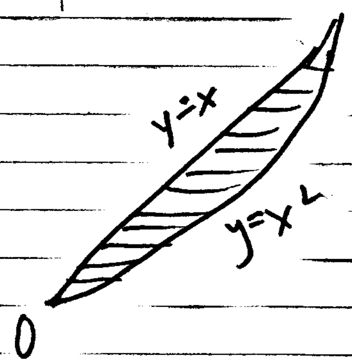
not an area

$$\int_0^{\pi} |\cos x - \sin x|\, dx$$



$$\int_0^1 (x - x^2)\,dx$$

$$= x^2/2 - x^3/3 \Big]_0^1 = \frac{1}{6}$$



$$\int_{y=0}^{y=1} [r(y) - l(y)]\,dy$$

$$= \int_{y=0}^{y=1} [\sqrt{y} - y]\,dy$$

$$= \frac{y^{3/2}}{3/2} - y^2/2 \Big|_0^1$$

$$= 2/3 - \frac{1}{2} = \boxed{\frac{1}{6}}$$

$\boxed{\text{EMBARRASSING}}$

Area of semicircle of radius 1

$$\int_{-1}^1 \sqrt{1-x^2}\,dx$$

Slices   CROSS SECTION has area $A(x)$

Volume of piece $dV = A(x)\,dx$

Volume between $x = x_1$ and $x = x_2$

$$\int_{x_1}^{x_2} A(x)\,dx$$

CIRCULAR CROSS SECTIONS

$$A(x) = \pi \cdot \left[\text{radius}(x)\right]^2$$
$$= \pi\left[f(x)\right]^2$$

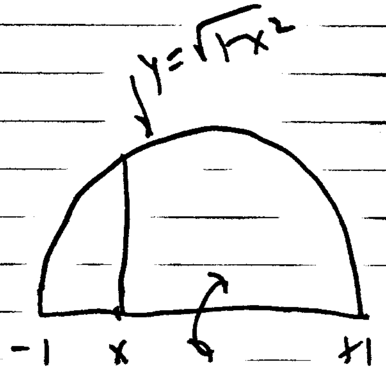
HOLE THRU MIDDLE: $f(x)$ top

$g(x)$ bottom

HOLE $\displaystyle\int_{x_1}^{x_2} \pi \cdot (f(x))^2\,dx - \int_{x_1}^{x_2} \pi \cdot (g(x))^2\,dx$

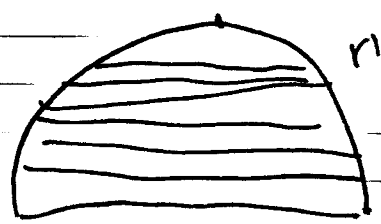
SPHERE top: $y = \sqrt{1-x^2}$

$x = -1$ to $x = +1$



$$\int_{x=-1}^{x=+1} \pi \left(\sqrt{1-x^2}\right)^2 dx$$

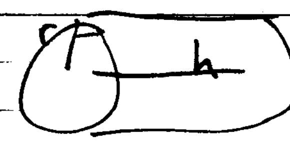
$$= \int_{-1}^{1} \pi \cdot (1-x^2)\,dx = \boxed{\tfrac{4}{3}\pi}$$



right: $x = \sqrt{1-y^2}$

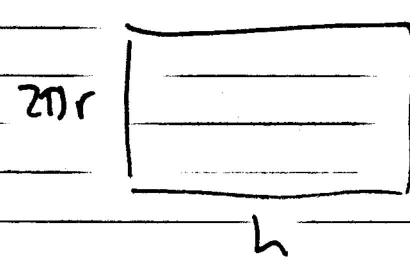
thickness dy

left $x = -\sqrt{1-y^2}$



radius

$$dV = 2\pi \cdot r(y) \cdot h(y)\,dy$$



$$= 2\pi y \left[2\sqrt{1-y^2}\right] dy$$

sphere

$$\int_{y=0}^{y=1} 2\pi\, y \cdot 2\sqrt{1-y^2}\, dy \rightarrow \text{let } u = 1-y^2$$

$$du = -2y\,dy$$

$$= \int -2\pi \; u^{1/2} \; du$$

$$= -2\pi \frac{u^{3/2}}{3/2} = -\frac{4\pi}{3} u^{3/2}$$

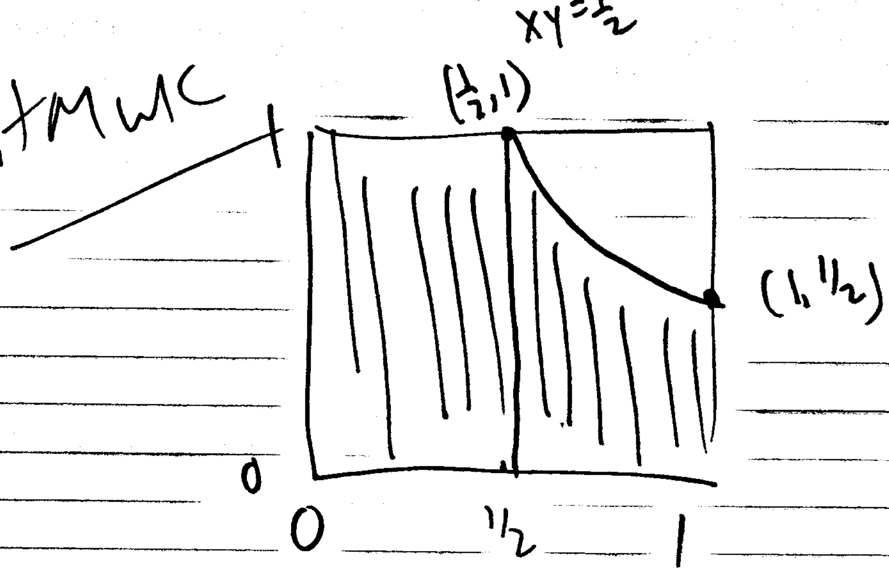
$y=0,\ u=1$

$y=1,\ u=0$

$$-\tfrac{4}{3}\pi\, u^{3/2} \Big|_{u=1}^{u=0}$$

$$\cancel{\int_{y=0}} \quad -\frac{4\pi}{3}\left(1-y^2\right)^{3/2} \Big|_{y=0}^{y=1} = \boxed{\frac{4\pi}{3}}$$

HMWC



$$xy < \frac{1}{2} \quad \text{vs.} \quad xy > \frac{1}{2}$$

ANS: $\frac{1}{2} + \int_{x=1/2}^{x=1} \frac{1}{2x}\,dx$