# 6.8 EXPONENTIAL MODELS

$$\frac{dy}{y} = k\,dt$$

OR

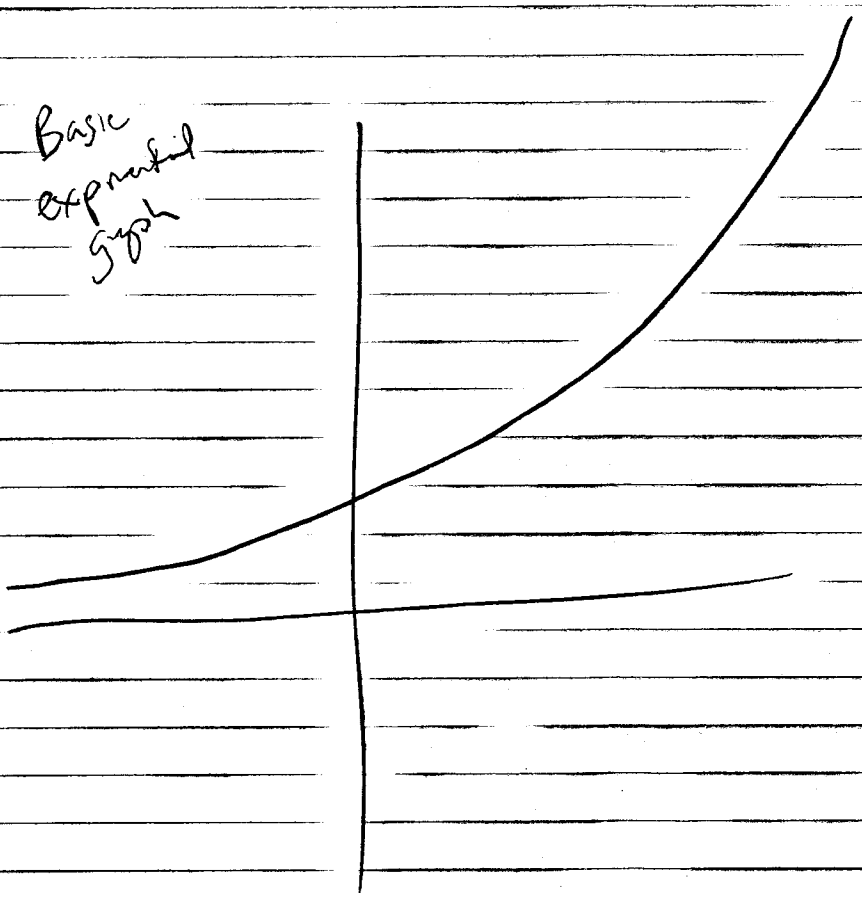
$$\frac{dy}{dt} = ky$$

$k=1$ $\quad \frac{dy}{dt} = y \rightarrow y = e^t$

IN FACT $y = Ce^t$

Put k as "constant" but unknown

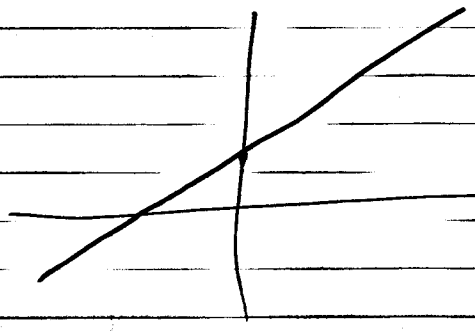
$$y = Ce^{kt}$$




IF $y = Ce^{kt}$ $\quad C > 0$

then $\ln y = \ln C + \ln(e^{kt})$

$= \ln C + kt$



Linear plot $\ln y$ vs $t$

[SEMILOG PLOT]

Alternate way to do growth problems (Book, example 2)

$(6.0) \cdot \left(\frac{6.9}{6.0}\right)^{t/10}$ ← 10 yr "chunks"

↓ C

↑ growth factor (multiplicative)

Doubling $2^{t/T_2}$

Book, Exercise 14

$Ce^{kt}$ = # of cells after

$t$ = # weeks to get 1500 cells

$Ce^{kt} = 1500$

$t=0$ $\quad C = 8$

when $t=6$ $\quad Ce^{k \cdot 6} = 2C$

$e^{6k} = 2$

$e^{k} = 2^{1/6}$

$6k = \ln 2$, $k = \frac{1}{6}\ln 2$

Solve $e^{kt} = 1500/8$

So $kt = \ln(1500/8)$

$$t = \frac{\ln(1500/8)}{\frac{1}{6}\ln(2)}$$

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Interest / growth (rate) — example 2%

$$C\left(1.02\right)^{t}$$

base: Multiplicative factor is $1 +$ "interest rate" $p$ as decimal

Use 2% → Factor of 1.02 per year