Goal: You will use Maple to gain a better understanding of numerical integration.

Commands: The main Maple commands you will use are leftsum, middle-
sum, rightsum, trapezoid and simpson. These are all quite similar in syntax
and fairly easy to use. However, Maple can be picky about how you
type things, so be careful.

Assignment: Your assignment is to compute numerical approximations to a
simple integral in order to see how the various methods behave as the number
of subintervals increases. In particular, you will observe the behavior quoted
in our book for the error.

The integral we will look at is

$$\int_{0}^{6} x^5 \,dx = \frac{1}{6} x^6|_{0}^{6} = 7776.$$ 

Directions: Use the Maple commands to find the numerical values of the
approximations for 20, 40 and 80 subintervals. This doubles the number of
points each time. According to the theory, the error should be reduced by
1/2 each time for left and right endpoints, 1/4 for midpoint and trapezoid,
and by 1/16 for Simpson.

The Maple commands you should use include:

1. To find an integral of a function f: \texttt{int(f(x),x=a..b)};
2. To load the student package, which has built-in commands: \texttt{with(student)};
3. To find the left endpoint approximation in n steps: \texttt{leftsum(f(x),x=a..b,n)};
4. To find the right endpoint approximation in n steps: \texttt{rightsum(f(x),x=a..b,n)};
5. To find the midpoint approximation in n steps: \texttt{midpoint(f(x),x=a..b,n)};
6. To find the trapezoid approximation in n steps: `trapezoid(f(x), x=a..b, n);`

7. To find Simpson’s approximation in n steps: `simpson(f(x), x=a..b, n);`

8. To convert some of the above outputs into decimals: `evalf(value(%));`

You can combine the above as needed, so for example you might type 
`evalf(value(simpson(f(x), x=a..b, n)));`

Your write-up should of course be clear, well-written, and should have the following elements:

1. A table of the numerical results for the various number of subdivisions and the type of approximation;

2. A brief discussion of the results – were the errors decreasing more or less as claimed;

3. A numerical approximation to the appropriate constant in the error formulas for the trapezoid and Simpson’s rule as in the text using your values for \( n = 80 \), which are both called \( M \).

See the maple sample and the maple introduction from the course web page if you want further information.