MATH 114 — PROF SACHS — HOMEWORK #1 SOLUTION

(1) $\int \frac{\ln x}{x}\,dx$ let $u = \ln x$, $du = \frac{1}{x}dx$

so we get $\int u\,du = \frac{1}{2}u^2 + C$ $\boxed{\text{ANS } \frac{1}{2}(\ln x)^2 + C}$

(2) $\frac{d}{dx}\ln(g(x)) = \frac{1}{g(x)} \cdot \frac{d}{dx}g(x)$ [Chain rule: $\ln u$, $u = g(x)$]

$= \frac{g'(x)}{g(x)}$

(a) $\frac{d}{dx}\ln(x+4) = \frac{1}{x+4}$ (b) $\frac{d}{dx}\ln((x+9)^2) = \frac{1}{(x+9)^2}\cdot 2(x+9)$

$= \frac{2}{x+9}$ [could use $\ln((x+9)^2) = 2\ln(x+9)$ also]

(c) $\frac{d}{dx}\ln[(x+4)(x+9)]$

$= \frac{1}{(x+4)(x+9)}[1\cdot(x+9) + (x+4)\cdot 1]$

$= \frac{1}{x+4} + \frac{1}{x+9}$ (or) $= \frac{2x+13}{(x+4)(x+9)}$

(3) [#29, Section 6.7] (a) ~~true~~ — basic property of ln [see Theorem 6.4]

(b) false — $\ln 1 = 0$ but $0 \neq e^y$ for any y and certainly not $y=1$

(c) False $\ln(x+y) \neq \ln(x) + \ln(y)$ in general — example $x = y = e$

$\ln(2e) \neq 1 + 1 = 2$ [area from 1 to 2e = area from 1 to e + area from e to 2e

area < 1 → height 1/e at left, base: e]

(d) False $2^x = (e^{\ln 2})^x = e^{x \ln 2}$ but $x \ln 2$ is not same function as $2 \ln x$.

$(\ln(x^2) = 2 \ln x)$

(e) False area under $y = 1/x$ starts from $x=1$ NOT $x=0$

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(4) (Prob. 60, Sect. 6.7) $\frac{d}{dx} \ln(xy) = \frac{1}{xy} \cdot \frac{d}{dx}(xy) = \frac{1}{xy} \cdot y = \frac{1}{x}$

so $\frac{d}{dx} \ln(xy) = \frac{d}{dx} \ln(x) = \frac{1}{x}$ — For all $x > 0$, $\ln(xy) = \ln(x) + C$

At $x = 1$ we get $C = \ln(y)$ so for $x, y > 0$, $\ln(xy) = \ln(x) + \ln(y)$.

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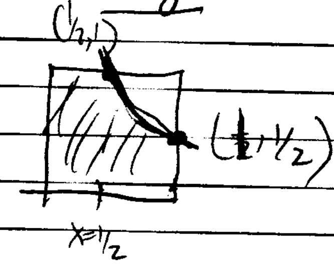
(5) (Prob 55, Sect 6.7) From text hint, random x, y leads to area $\frac{1}{2}xy$ so want $\frac{1}{2}xy < 1/4$ or $xy < \frac{1}{2}$ [note: probability equally likely, all x, y in square $\Leftrightarrow$ area 1, so no need to average after integrating. Now graph $xy < 1/2$ or $y < \frac{1}{2} \cdot \frac{1}{x}$



For $x \leq 1/2$, $xy < 1/2$ but if $x > \frac{1}{2}$, need to integrate. So we get $\frac{1}{2} + \int_{1/2}^{1} \frac{1}{2x}\,dx$

$$= \frac{1}{2} + \frac{1}{2}\ln x \Big|_{1/2}^{1} = \frac{1}{2} + \frac{1}{2}\ln(1) - \frac{1}{2}\ln(1/2) = \frac{1}{2} + \frac{1}{2}\ln(2)$$

$$\approx \boxed{0.846574}$$

(6) (#25, Sect. 6.8) (a) NO - After 1 yr, 1.06 y_0 this is not $e^{0.06} \approx 1.06184$

(b) NO $(1.10)^3 = 1.331$ (compounding effect) continuous compounding effect

(c) YES Careful wording "one third of its current amount" is useful

(d) YES Faster growth $\Leftrightarrow$ shorter doubling time

(e) YES $e^{kT} = 10$ for T means $e^{k(t+T)} = e^{kt}e^{kT} = e^{kT}e^{kt}$
$= 10e^{kt}$

(7) (#39, Sect. 6.8) $y_0 e^{kt}$, $y_0(1+r)^t$, $y_0 2^{t/T_2}$ linked as follows

$$(1+r)^t = e^{kt} = 2^{t/T_2}$$ take ln of all

$$t\ln(1+r) = kt = t/T_2 \ln(2)$$

so $k = \ln(1+r) = \dfrac{\ln(2)}{T_2}$ solved for k - ok to stop here

or

solve the other ways.

$1+r = e^k = 2^{1/T_2}$ so $r = e^k - 1 = 2^{1/T_2} - 1$

and $T_2 = \dfrac{\ln(2)}{k} = \dfrac{\ln(2)}{\ln(1+r)}$