HOMEWORK 7 SOLUTION

Math 113 – Fall 2004 – Prof. SACHS

3.2

#3) $f(x) = \sin^{-1} x$ on $[-1, 1]$ — f is continuous on $[-1,1]$, differentiable on $(-1,1)$

$\Rightarrow f'(c) = \dfrac{\sin^{-1}(1) - \sin^{-1}(-1)}{2} = \dfrac{\pi}{2}$; since $f'(c) = \dfrac{1}{\sqrt{1-c^2}}$ this says

$\sqrt{1-c^2} = \frac{2}{\pi}$; $1 - c^2 = 4/\pi^2$; $c^2 = 1 - 4/\pi^2$, $\boxed{c = \pm\sqrt{1 - 4/\pi^2} \approx \pm .77118}$

#4) f is continuous on $[2,4]$, differentiable also; $\dfrac{\ln(4-1) - \ln(2-1)}{4-2} = \dfrac{\ln(3)}{2}$

$f'(c) = \dfrac{1}{c-1} = \dfrac{\ln(3)}{2}$ so $c - 1 = \dfrac{2}{\ln 3}$ and $\boxed{c = 1 + \dfrac{2}{\ln(3)}}$

so $\boxed{c \approx 2.82048}$

#14) $g'(x) = \frac{1}{x} + 2x \Rightarrow g(x) = \ln x + x^2 + C$; for $x=1$, $g(1) = -1$

so $\ln(1) + 1^2 + C = -1$; $C = -2$: $\boxed{g(x) = \ln x + x^2 - 2}$

#22) $a = 9.8$, $v(0) = -3$, $s(0) = 0$

$v(t) = 9.8t + v(0) \Rightarrow v(t) = 9.8t - 3$

$\Rightarrow s(t) = 4.9t^2 - 3t + s(0) = 4.9t^2 - 3t$

#23) $a = -4\sin 2t$, $v(0) = 2$, $s(0) = -3$

$v(t) = 2\cos 2t + C$: $t=0$, $v(0) = 2 \Rightarrow C = 0$

$v(t) = 2\cos 2t$ so $s(t) = \sin 2t + s(0)$ since $\sin 0 = 0$

Get $\boxed{s(t) = \sin 2t - 3}$

$\boxed{3.3}$ **#8)** Estimating: f decreasing: $-2.1 \le x < -1$ ($f' < 0$)
$0 < x < 1.2$

$f' = 0$: about $x = -1, 0, 1.2$ (would be 1 and could be -1.2)

$f' > 0$ (increasing) $-1 < x < 0$ and $1.2 < x \le 2.1$

f'' is positive for $-2.1 \le x < -.6$ and $.6 < x \le 2.1$

$f'' = 0$ (inflection points) $\approx \pm 0.6$

f'' is negative for $-.6 < x < .6$

#10) f increasing ($f' > 0$) $-2 < x < 2$
f decreasing ($f' < 0$) $x < -2$ and $x > 2$

Extreme values: $x = -2$ (local min)
$x = +2$ (local max)

[$x = 0$ is neither max nor min.]

#14) (a) crit. pts ($f' = 0$) $x = 1$, $x = -2$
(b) $f' > 0$: $x > -2$ except $f' = 0$ at $x = 1$ INCREASING
$f' < 0$: $x < -2$ DECREASING
(c) $x = -2$: local min; no local max

#18) $y = -2x^3 + 6x^2 - 3$, $y' = -6x^2 + 12x$
$y'' = -12x + 12$ $\rightarrow$ we conclude $y' = 0 \Leftrightarrow -6x(x-2) = 0$

$x = 0$, $x = 2$; $y' > 0$ if $0 < x < 2$; $y' < 0$ if $x < 0$ or $x > 2$

So y is (a) increasing: $0 < x < 2$
(b) decreasing: $x < 0$ or $x > 2$

(c) concave up : $x<1$, (d) concave down $x>1$

We get: (e) local extreme — min at $x=0$, max ~~at~~ 2
value: -3 , value: 13

(f) inflection pt at $x=1$

$\boxed{\#34}$ $y'=(x-1)^2(x-2)(x-4)$: (a) local max when y' goes from $+$ to $-$ only at $x=2$

(b) local min when y' changes sign from $-$ to $+$: only at $x=4$

(c) Inflection: $y''=2(x-1)(x-2)(x-4)+(x-1)^2(x-4)+(x-1)^2(x-2)$

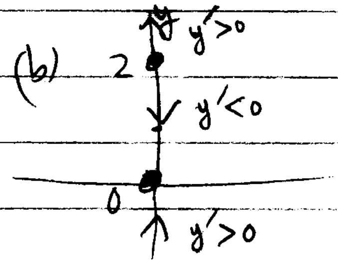
$=(x-1)\left[2(x-2)(x-4)+(x-1)(x-4)+(x-1)(x-2)\right]$

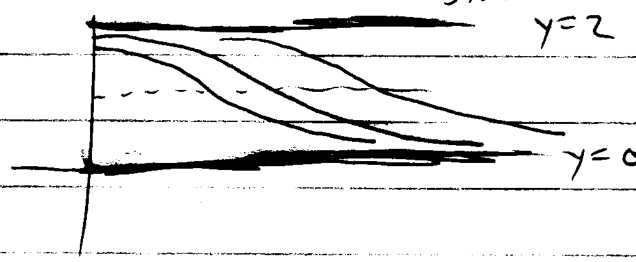
$=(x-1)\{2(x^2-6x+8)+x^2-5x+4+x^2-3x+2\}$

$=(x-1)\{4x^2-20x+22\}$ - roots $x=1, \frac{5\pm\sqrt{3}}{2}$

inflection points.

$\boxed{3.4}$ $\boxed{\#4}$ $\frac{dy}{dx}=y^2-2y=y(y-2)$ — (a) equilibria: $y=0$, $y=2$
stable, unstable

(b) 

(c) 

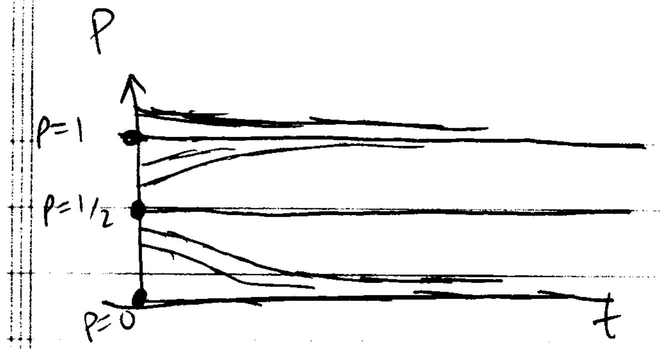
$y''=\frac{d}{dx}\left[y^2-2y\right]=2y\frac{dy}{dx}-2\frac{dy}{dx}=(2y-2)\underbrace{(y)(y-2)}_{\text{neg. between } 0,2}$

0 at $y=1$.

$\boxed{\#12}$ $\frac{dP}{dt}=3P(1-P)(P-1/2)$ 0 at $P=0, 1, 1/2$

$\frac{dP}{dt}>0$ for $P<0$ or $\frac{1}{2}<P<1$

$\frac{dP}{dt}<0$ for $0<P<1/2$ or $P>1$



#19) $L\frac{di}{dt} + Ri = V \Rightarrow \frac{di}{dt} = \frac{V - Ri}{L}$

$\frac{di}{dt} > 0$ if $i < V/R$, $\frac{di}{dt} < 0$ if $i > \frac{V}{R}$

if $i = V/R$, solution is equilibrium.

As $t \to \infty$, all solutions tend to equilibrium (steady state)

value $\underline{V/R}$