HOMEWORK 6 SOLUTIONS
MATH 113 - Fall 2004 - Prof Sachs

$\boxed{2.9}$ #8) $y = x^2e^x - xe^x$; $\dfrac{dy}{dx} = \underset{\uparrow\,\text{product rule}}{2xe^x + x^2e^x} - \underset{\uparrow\,\text{again}}{e^x - xe^x}$

$= x^2e^x + xe^x - e^x$ [organize by powers of x]

#23) $y = \ln(1/x) = \ln(u)$ where $u = 1/x = x^{-1}$

$\dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dx} = \dfrac{1}{u}\cdot(-1)\cdot x^{-2} = -\dfrac{1}{u\cdot x^2}$ but $u\cdot x = 1 \Rightarrow \boxed{-\dfrac{1}{x}}$

Note! $\ln(1/x) = -\ln(x)$ also; derivative is consistent with that.

#26) $y = \ln(2x+2)$: $u = 2x+2$, $y = \ln u \Rightarrow \dfrac{dy}{dx} = \dfrac{dy}{du}\cdot\dfrac{du}{dx} = \dfrac{1}{2x+2}\cdot 2$

Note: $2x+2 = 2\cdot(x+1)$ so $\ln(2x+2) = \ln(2) + \ln(x+1)$ $= \dfrac{1}{x+1}$

(consistent again)

#49) $A = 20\cdot(1/2)^{t/140}$: t is in days, A is in grams

Chain rule: $\dfrac{dA}{dt} = \dfrac{dA}{du}\dfrac{du}{dt}$, $A = 20\cdot\left(\frac{1}{2}\right)^u$, $u = t/140$

Rewrite A as $20\cdot(1/2)^u = 20\cdot e^{\ln(1/2)\cdot u} = 20e^v$, $v = \ln(1/2)u$.

Then $\dfrac{dA}{dt} = \dfrac{dA}{du}\dfrac{du}{dt} = \dfrac{dA}{dv}\dfrac{dv}{du}\dfrac{du}{dt} = 20e^v\cdot\ln(1/2)\cdot\dfrac{1}{140}$

When $t = 2$ days, we get $\dfrac{dA}{dt} = 20e^{\frac{2}{140}\ln(1/2)}\cdot\ln(1/2)\cdot\dfrac{1}{140} \approx -.098$ grams/day

$\boxed{\text{Chap 2 Practice}}$

#4) $y = (2x-5)(4-x)^{-1}$ product + chain (a) or quotient (b)

(a) $2\cdot(4-x)^{-1} + (2x-5)(-1)(4-x)^{-2}\cdot(-1) = 2\cdot(4-x)^{-1} + (2x-5)(4-x)^{-2}$

$= \dfrac{2\cdot(4-x) + 2x-5}{(4-x)^2} = \dfrac{3}{(4-x)^2}$

(b) $\dfrac{2\cdot(4-x) - (2x-5)(-1)}{(4-x)^2} = \dfrac{3}{(4-x)^2}$

#10) $s=\cos^4(1-2t)$ chain rule $s=[\cos(1-2t)]^4=u^4$, $u=\cos(1-2t)$

$\frac{ds}{dt}=\frac{ds}{du}\frac{du}{dt}=4u^3\cdot\frac{du}{dt}$. Do $\frac{du}{dt}$ with chain rule again: $u=\cos w$, $w=1-2t$

Find $\frac{du}{dt}=-\sin w\cdot\frac{dw}{dt}=-\sin w\cdot(-2)=2\sin(1-2t)$. So $\frac{ds}{dt}=\boxed{8\cos^3(1-2t)\sin(1-2t)}$

#25) $y=\frac{1}{4}xe^{4x}-\frac{1}{16}e^{4x}$, $\frac{dy}{dx}=\underbrace{\frac{1}{4}e^{4x}+\frac{1}{4}x\cdot\frac{d}{dx}(e^{4x})}_{\text{product rule}}\underbrace{-\frac{1}{16}\frac{d}{dx}(e^{4x})}_{\text{chain rule (easy)}}$

so $\frac{dy}{dx}=\frac{1}{4}e^{4x}+\frac{1}{4}x\cdot 4e^{4x}-\frac{1}{16}\cdot 4e^{4x}=\boxed{xe^{4x}}$

#60) (a) $(x^{1/2}f(x))'=\frac{1}{2}x^{-1/2}\cdot f(x)+x^{1/2}\cdot f'(x)$ [product rule; power rule]

using $x=1$, get $\frac{1}{2}\cdot 1\cdot(-3)+1\cdot\frac{1}{5}=-\frac{3}{2}+\frac{1}{5}=\boxed{-\frac{13}{10}}$

(b) $(\sqrt{f(x)})'=\frac{1}{2}\cdot(f(x))^{-1/2}\cdot f'(x)$ ← CHAIN RULE! at $x=0$, get

$\frac{1}{2}\cdot(9)^{-1/2}\cdot(-2)=\boxed{-\frac{1}{3}}$

(c) $[f(x^{1/2})]'=f'(x^{1/2})\cdot\frac{1}{2}\cdot x^{-1/2}$: at $x=1$ get $f'(1)\cdot\frac{1}{2}=\boxed{\frac{1}{10}}$

(d) $[f(1-5\tan x)]'$ at $x=0$: $f(u)$, $u=1-5\tan x$ → $f'(u)\cdot\frac{du}{dx}$

At $x=0$, $u=1$ and $\frac{du}{dx}=-5\sec^2 x=-5$ at $x=0$: Answer is $\frac{1}{5}(-5)=\boxed{-1}$

(e) $\left(\frac{f(x)}{2+\cos x}\right)'=\frac{f'(x)\cdot(2+\cos x)-f(x)\cdot(-\sin x)}{(2+\cos x)^2}$: at $x=0$, $\cos x=1$

$f(0)=9$

$f'(0)=-2$

Get value: $\frac{-2\cdot(3)+0}{3^2}=\boxed{-\frac{2}{3}}$

(f) $\left(10\sin\left(\frac{\pi x}{2}\right)\cdot f^2(x)\right)'$ at $x=1$: product rule, etc. gives

$\underbrace{10\cdot\cos\left(\frac{\pi x}{2}\right)\cdot\frac{\pi}{2}}_{\text{chain rule}}\cdot[f(x)]^2+10\sin\left(\frac{\pi x}{2}\right)\cdot 2f(x)\cdot f'(x)$: set $x=1$

$f(1)=-3$ | $\sin(\pi/2)=1$

$f'(1)=\frac{1}{5}$ | $\cos(\pi/2)=0$

Get $0+10\cdot 1\cdot 2\cdot(-3)\cdot\frac{1}{5}=\boxed{-12}$

#89) Ⓐ is derivative of Ⓑ: It can't be the other way since near $x=0$, slope of Ⓐ is negative (indeed everywhere) while Ⓑ is positive near 0.

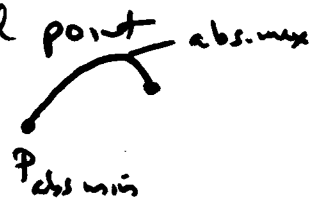
#93) (a) Derivative is 0 at largest/smallest value of function
(b) Function is <u>1700</u> when derivative is biggest (40)
Function is about <u>1300</u> when derivative is smallest (-53)

$\boxed{3.1}$ #3) Absolute max at $x=c$; no abs. min [endpoints missing]
#4) neither abs. max, min – [hole plus endpoints missing]

#5) Abs. max at c, abs min at a.
#6) Abs min at c, abs max at a.

#15) $f' = 2/3$ – never 0 (line) $\Rightarrow$ max, min at endpts.
$f(-2) = -4/3 - 5 = -19/3$, $f(3) = \frac{2}{3}\cdot 3 - 5 = -3$

#17) $f(x) = 4 - x^2$, $f'(x) = -2x$: $x=0$ is critical point
$f(0) = 4$, $f(-3) = 4 - 9 = -5$, $f(1) = 4 - 1 = 3$



#39) $f(x) = x(4-x^2)^{1/2}$
$f'(x) = 1\cdot(4-x^2)^{\frac{1}{2}} + x\frac{1}{2}(4-x^2)^{-1/2}\cdot(-2x)$ ← chain rule
Product rule
$= (4-x^2)^{1/2} - x^2\cdot(4-x^2)^{-1/2}$
$= \dfrac{4-x^2-x^2}{\sqrt{4-x^2}} = \dfrac{4-2x^2}{\sqrt{4-x^2}}$

Domain: $-2 \le x \le 2$; only local extremes are $x^2 = 2$
<u>or</u> $x = \pm\sqrt{2}$: At $\sqrt{2}$, $y = \sqrt{2}\cdot\sqrt{2} = 2$ – local max
At $-\sqrt{2}$, $y = -\sqrt{2}\cdot\sqrt{2} = -2$ – local min

<u>Note</u>: For $-2 < x < -\sqrt{2}$, $f' < 0$;
For $-\sqrt{2} < x < +\sqrt{2}$, $f' > 0$;
For $\sqrt{2} < x < 2$, $f' < 0$