HOMEWORK #5 – SOLUTIONS
MATH 113 – Fall 2004 – Prof SACHS

Section 2.5 **#14** $s = \sin\left(\frac{3\pi t}{2}\right) + \cos\left(\frac{3\pi t}{2}\right)$ – chain rule

$u = \frac{3\pi t}{2}$, $s = \sin(u) + \cos(u)$ so $\frac{ds}{dt} = \frac{ds}{du}\frac{du}{dt}$

$= \left[\cos\left(\frac{3\pi}{2}t\right) - \sin\left(\frac{3\pi t}{2}\right)\right]\left(\frac{3\pi}{2}\right)$

#52 (a) $5f'(1) - g'(1) = 5(-1/3) - (-8/3) = 1$

(b) $(f(x)g^3(x))' = f'(x)\cdot(g(x))^3 + f(x)([g(x)]^3)'$ PRODUCT RULE

$= f'(x)(g(x))^3 + f(x)\cdot 3[g(x)]^2\cdot g'(x)$ CHAIN/POWER

At $x=0$: $f'(0)\cdot(g(0))^3 + 3f(0)(g(0))^2\cdot g'(0) = 5\cdot 1^3 + 3\cdot 1\cdot 1^2\cdot 1/3 = 6$

(c) $\left(\frac{f(x)}{g(x)+1}\right)' = \frac{f'(x)(g(x)+1) - f(x)(g'(x))}{(g(x)+1)^2}$; at $x=1$, $\frac{[1/3\cdot(-3)] - 3(\frac{8}{3})}{(-3)^2}$

QUOTIENT

$= \frac{1+8}{9} = 1$

(d) $[f(g(x))]' = f'(g(x))\cdot g'(x)$ so at $x=0$, $g(0)=1$

CHAIN RULE

get $f'(1)\cdot g'(0) = (-1/3)(\frac{1}{3}) = -\frac{1}{9}$

(e) $[g(f(x))]'$ at $x=0$: $g'(f(0))\cdot f'(0) = g'(1)f'(0) = -\frac{40}{3}$

(f) $(x^{11} + f(x))^{-2}$ at $x=1$: $u = x^{11} + f(x)$, $y = u^{-2}$

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = -2\cdot u^{-3}\cdot(11\cdot x^{10} + f'(x))$

POWER/CHAIN

$x=1$, $u = 1^{11} + f(1) = 1+3 = 4$; $-2\cdot(4)^{-3}\cdot[11 + (-1/3)]$

$= \frac{-2}{64}\left(\frac{32}{3}\right) = -\frac{1}{3}$

(g) $y = f(x+g(x))$: $f(u)$, $u = x+g(x)$: $\frac{dy}{du}\cdot\frac{du}{dx} = f'(u)\cdot(1+g'(x))$

At $x=0$, $u = 0 + g(0) = 1$; $f'(1)(1+g'(0)) = -1/3 \cdot (4/3) = -4/9$

#56 $y = x^{3/2}$ as (a) $y = u^3$, $u = x^{1/2}$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 3u^2 \cdot \frac{1}{2}x^{-1/2}$

$= \frac{3}{2}x^{+1/2}$

(b) $y = u^{1/2}$, $u = x^3$: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{2}u^{-1/2} \cdot 3x^2 = \frac{3}{2}x^{-3/2} \cdot x^2 = \frac{3}{2}x^{1/2}$

#60 (a) $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, $u = \frac{2\pi}{365}(x-101)$, $y = 37\sin u + 25$

so $\frac{dy}{dx} = 37\cos u \cdot \frac{2\pi}{365} = \frac{37 \cdot 2\pi}{365}\cos\left(\frac{2\pi}{365}(x-101)\right)$

cos is biggest at angle $0 \Leftrightarrow x = 101$ day for max.

(b) About .64 [value of $\left(\frac{37 \cdot 2\pi}{365}\right)$] degrees/day

$\boxed{\text{Section 3.6}}$ #18 $y = \frac{2\sqrt{x}}{3(1+\sqrt{x})}$ chain rule/power/quotient

$$dy = \frac{2 \cdot \frac{1}{2} x^{-1/2} \cdot 3 \cdot (1+\sqrt{x}) - \cancel{2}\sqrt{x} \cdot 3 \cdot (\frac{1}{2}x^{-1/2})}{[3(1+x^{1/2})]^2}\, dx$$

$$= \frac{3x^{-1/2}(1+x^{1/2}) - 3}{9 \cdot (1+x^{1/2})^2}\, dx$$

DON'T FORGET

$$= \frac{1}{3x^{1/2}(1+x^{1/2})^2}\, dx$$

NOTE: $(1+x^{1/2})^2$
$= 1 + 2x^{1/2} + (x^{1/2})^2$
$= 1 + 2x^{1/2} + x$
$\neq 1 + x$

#29) $dV = \frac{4}{3}\pi 3a^2\, dr = 4\pi a^2\, dr$

#30) $dS = 4\pi \cdot (2a)\, dr$ as $a \to a + dr$

Section 2.8

#19) (a) $y = \tan x$ at $(\pi/4, 1)$

$\frac{dy}{dx} = \sec^2 x$ at $x = \pi/4 \Leftrightarrow \cos^2(\pi/4) = \frac{1}{2}$

so $\sec^2(\pi/4) = 2$

$y - 1 = 2(x - \pi/4)$

(b) $y = \tan^{-1} x$: using formula on p 219, $\frac{dy}{dx} = \frac{1}{1+x^2}$; $x=1$ get $1/2$

$y - \pi/4 = \frac{1}{2}(x-1)$ INVERSE FUNCTION

#20) (a) $f(1) = 1^5 + 2 \cdot 1^3 + 1 - 1 = 3$, $f'(1) = 5 \cdot 1^4 + 6 \cdot 1^2 + 1 = 12$

(b) $f^{-1}(3) = 1$, $(f^{-1})'(3) = \frac{1}{f'(1)} = \frac{1}{12}$

#21) (a) Since $f'(x) = -\sin x + 3 \geq 2$, f^{-1} is sensible and differentiable

(b) $f(0) = \cos(0) + 3 \cdot 0 = 1$, $f'(0) = -\sin(0) + 3 = 3$

(c) $f^{-1}(1) = 0$, $(f^{-1})'(1) = \frac{1}{f'(0)} = \frac{1}{3}$