Math 113 – Prof. Sachs – Solution to HMWK 4

$\boxed{2.1}$ **#2)** From definition, $g'(t) = \lim_{h\to 0} \frac{g(t+h) - g(t)}{h}$

$$= \lim_{h\to 0} \frac{\frac{1}{(t+h)^2} - \frac{1}{t^2}}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{(t+h)^2} - \frac{1}{t^2}\right] = \lim_{h\to 0} \frac{1}{h}\left[\frac{t^2 - (t+h)^2}{(t+h)^2 t^2}\right]$$

$$= \lim_{h\to 0} \frac{1}{h} \cdot \frac{t^2 - (t^2 + 2th + h^2)}{(t+h)^2 \cdot t^2} = \lim_{h\to 0} \frac{1}{h} \frac{-2th - h^2}{(t+h)^2 t^2}$$

$$= \lim_{h\to 0} \frac{-2t - h}{(t+h)^2 t^2} = \frac{-2t}{t^4} = \frac{-2}{t^3}$$

so $g'(-1) = \frac{-2}{(-1)^3} = 2$, $g'(2) = \frac{-2}{2^3} = -1/4$

#4) $f(x) = x + 9/x$ so $f'(x) = \lim_{h\to 0} \frac{1}{h}\left(x+h + 9/(x+h) - x - 9/x\right)$

$$= \lim_{h\to 0} \frac{1}{h}\left(h + \frac{9}{x+h} - 9/x\right)$$

$$= \lim_{h\to 0} \frac{h}{h} + \lim_{h\to 0} \frac{1}{h}\left[\frac{9x - 9(x+h)}{(x+h)x}\right] = 1 - \frac{9}{x^2}$$

so $f'(-3) = 1 - \frac{9}{(-3)^2} = 0$

#8) $s = 5t^3 - 3t^5$, $s'(t) = 15t^2 - 15t^4$, $s''(t) = 30t - 60t^3$
using POWER RULE, etc.

#13) (a) $y' = 3x^2 - 4$; at $(2,1)$, slope is $3 \cdot 2^2 - 4 = 12 - 4 = 8$
eqn $y - 1 = 8(x-2)$

(b) slope is $3x^2-4$ so range is $[-4,\infty)$

(c) when slope is 8: $3x^2-4=8$ so $3x^2=12$, therefore $x=\pm 2$
at $x=2$, $y=8-8+1=1 \Rightarrow$ tangent eqn $y-1=8(x-2)$
at $x=-2$, $y=-8+8+1=1 \Rightarrow$ tangent eqn $y-1=8(x+2)$

#15-18 15: f_1' should be negative for $x<0$, positive for $x>0$ so it must be (b)

16: f_2' should be positive for $x<0$, 0 at 0, positive for $x>0$ so it should be (a)

17: f_3' goes back and forth – 0 at $x=0$, posi just to left, negative just to right and so forth – must be (d)

18: f_4' is negative until value <0 where graph bottoms out then positive until $x=0$, then negative until bottoms out then positive – must be (c)

#33 $y=2x^2-13x+5 \Rightarrow \frac{dy}{dx}=4x-13$: $4x-13=-1$ if $4x=12$ so $x=3$

Slope -1, $y=2\cdot 9-13\cdot 3+5=18-39+5$
$=-16$

$y+16=(-1)(x-3)$ is tangent line.

2.3 **#5** By product rule: $y'=\left[(3-x^2)(x^3-x+1)\right]'$

$$=(3-x^2)'\cdot(x^3-x+1)+(3-x^2)(x^3-x+1)'$$
$$=(0-2x)\cdot(x^3-x+1)+(3-x^2)(3x^2-1)$$
$$=-2x^4+2x^2-2x-3x^4+9x^2+x^2-3$$
$$=-5x^4+12x^2-2x-3$$

Multiply out: $(3-x^2)(x^3-x+1) = 3x^3 - x^5 - 3x + x^3 + 3 - x^2$
$= -x^5 + 4x^3 - x^2 - 3x + 3$

derivative is: $-5x^4 + 12x^2 - 2x - 3$ ✓

#10) $v = (1-t)(1+t^2)^{-1} = \dfrac{1-t}{1+t^2}$ – Quotient:

$$v'(t) = \frac{(1-t)'(1+t^2) - (1-t)(1+t^2)'}{(1+t^2)^2} = \frac{(-1)(1+t^2)-(1-t)(2t)}{(1+t^2)^2}$$

$$= \frac{-1-t^2-2t+2t^2}{(1+t^2)^2} = \frac{-1-2t+t^2}{(1+t^2)^2}$$

#20)

(a) At $x=1$: $(uv)' = u'v + uv' = u'(1)v(1) + u(1)v'(1)$
$= 0.5 + 2\cdot(-1) = -2$

(b) $\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{u'v - uv'}{v^2} = \dfrac{0.5 - 2(-1)}{5^2} = \dfrac{2}{25}$

(c) $\dfrac{d}{dx}\left(\dfrac{v}{u}\right) = \dfrac{v'u - vu'}{u^2} = \dfrac{(-1)\cdot 2 - 0.5}{2^2} = \dfrac{-2}{4} = -1/2$

(d) $\dfrac{d}{dx}(7v - 2u) = 7v' - 2u' = 7\cdot(-1) - 2\cdot 0 = -7$

$\boxed{2.4}$ **#2)** $y = 3/x + 5\sin x$, $\dfrac{dy}{dx} = -3/x^2 + 5\cos x$

#8) $y = \dfrac{\cos x}{1+\sin x}$, $\dfrac{dy}{dx} = \dfrac{(\cos x)'\cdot(1+\sin x) - \cos x\cdot(1+\sin x)'}{(1+\sin x)^2}$

$$= \frac{-\sin x(1+\sin x) - \cos x\cdot\cos x}{(1+\sin x)^2}$$

(continued)

$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1+\sin x)^2} = \frac{-1-\sin x}{(1+\sin x)^2}$$

$$= \frac{-1}{1+\sin x}.$$

\#44) (a) $\sec x = \frac{1}{\cos x}$ so $\frac{d}{dx}(\sec x) = \frac{-1}{\cos^2 x} \cdot (\cos x)' = \frac{\sin x}{\cos^2 x}$

$$= \sec x \cdot \tan x$$

(b) $\csc x = \frac{1}{\sin x}$ so $\frac{d}{dx}\left(\frac{1}{\sin x}\right) = \frac{-1}{(\sin x)^2} \cdot (\sin x)' = \frac{-\cos x}{\sin^2 x}$

$$= -\csc x \cdot \cot x$$

(c) $\cot x = \frac{\cos x}{\sin x}$ so $\frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \frac{(\cos x)' \cdot \sin x - \cos x \cdot (\sin x)'}{(\sin x)^2}$

$$= \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$$

or $\cot x = \frac{1}{\tan x}$ so $\frac{d}{dx}\left(\frac{1}{\tan x}\right) = \frac{-1}{\tan^2 x} \cdot (\tan x)'$

$$= -\frac{1}{\tan^2 x} \cdot \sec^2 x = -\frac{1}{\frac{\sin^2 x}{\cos^2 x}} \cdot \frac{1}{\cos^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$$

Also $\frac{d}{dx}(\cos x \cdot \cos x) = (\cos x)' \cos x + (\cos x)(\cos x)'$

$$= -\sin x \cos x - \cos x \cdot \sin x$$
$$= -2 \sin x \cos x$$

and $\frac{d}{dx}(\sin x \sin x) = 2 \sin x \cdot (\sin x)' = 2 \sin x \cos x$

so sum is 0, which is good since $\sin^2 x + \cos^2 x = 1$ and derivative of 1 is 0.