MATH 113 – HOMEWORK 3 SOLUTIONS PROF. SACHS

SECTION 1.4

$\boxed{\text{pp. 132–134}}$

#14) Fails at $x+2=0$, $x=-2$ so $\frac{1}{(x+2)^2}+4$ is continuous for $x \neq -2 \Leftrightarrow (-\infty,-2) \cup (-2,\infty)$

#19) $s=\sqrt{2v+3}$ is continuous for $2v+3 \geq 0$
so $v \geq -3/2$ or in interval notation: $[-3/2, \infty)$

#26) $\cos x = x \Leftrightarrow x - \cos x = 0$: Since this is -1 at $x=0$ and $\pi - \cos\pi = \pi+1$ at $x=\pi$

there must a place in between where $x-\cos x=0$, since it is continuous.

#28) $x^3-8x+10$ takes on all y values since $\lim_{x\to\infty} f(x) = +\infty$, $\lim_{x\to-\infty} f(x) = -\infty$. In particular,

Since $f(-10) = (-10)^3 - 8(-10) + 10 = -1000 + 90 = -910$

and $f(10^3) = 10^9 - 8\cdot 10^3 + 10 > 900{,}000{,}000 = 10^9 - 10^8$,

all values in between are hit, including π, $-\sqrt{3}$ and $5{,}000{,}000$

#33) (a) $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

is discontinuous at every point. Either case at some point a, rational or irrational, using $\varepsilon = 1/3$, we have points in $(a-\delta, a+\delta)$ which land outside the target: $|f(x)-f(a)| = 1 > \frac{1}{3}$ since rational/irrational pairs are in all such intervals.

(b) Neither limit from right nor limit from left exists, so NO chance of right/left continuity.

#35) YES: If it did change sign we would have it be 0 in between points where $f>0$ and $f<0$.
INT. VALUE would include 0.

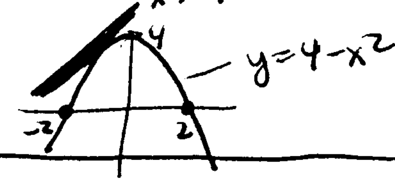
Section 1.5

$\boxed{\text{pp } 139-141}$ #1) Ator Near P_1 looks like about 1, near P_2 about 5

#3) Ator Near P_1, slope is about $5/2$; near P_2 about $-1/2$

#5) $y = 4 - x^2$: $\lim\limits_{x\to -1} \dfrac{4-x^2-3}{x-(-1)} = \lim\limits_{x\to -1} \dfrac{1-x^2}{x+1} = \lim\limits_{x\to -1} -(x-1) = +2$

Tangent line: $y - 3 = 2(x+1)$ Sketch:



#10) slope at $(1,4)$ of t^3+3t:

$\lim\limits_{t\to 1} \dfrac{t^3+3t-4}{t-1} = \lim\limits_{t\to 1} t^2+t+4 = 6$: $\boxed{\text{line } y-4 = 6(x-1)}$

OR $\lim\limits_{h\to 0} \dfrac{(1+h)^3+3(1+h)-4}{h}$

$= \lim\limits_{h\to 0} \dfrac{1+3h+3h^2+h^3+3+3h-4}{h}$

$= \lim\limits_{h\to 0} \dfrac{6h+3h^2+h^3}{h} = \lim\limits_{h\to 0} 6+3h+h^2 = 6$

#13) $\lim\limits_{x\to 3} \dfrac{\frac{1}{x-1} - \frac{1}{2}}{x-3}$

$= \lim\limits_{x\to 3} \dfrac{1}{x-3}\left[\dfrac{1}{x-1} - \dfrac{1}{2}\right]$

$= \lim\limits_{x\to 3} \dfrac{1}{x-3}\left(\dfrac{2-(x-1)}{(x-1)\cdot 2}\right)$

$= \lim\limits_{x\to 3} \dfrac{1}{x-3} \dfrac{(-1)(x-3)}{(x-1)\cdot 2} = \lim\limits_{x\to 3} \dfrac{-1}{(x-1)\cdot 2} = \boxed{-1/4}$

$\boxed{\text{OR}}$ $\lim\limits_{h\to 0} \dfrac{\frac{1}{(3+h)-1} - \frac{1}{2}}{h} = \lim\limits_{h\to 0} \dfrac{1}{h}\left[\dfrac{1}{2+h} - \dfrac{1}{2}\right] = \lim\limits_{h\to 0} \dfrac{1}{h}\left[\dfrac{2-(2+h)}{(2+h)\cdot 2}\right]$

$= \lim\limits_{h\to 0} \dfrac{-1}{(2+h)\cdot 2} = \underline{-1/4}$ ✓

$\boxed{\text{Practice, pp 142-3}}$ #3) (a) $3\cdot(-7) = -21$ (b) $(-7)^2 = 49$

(c) $(-7)\cdot 0 = 0$ (d) $\dfrac{-7}{0-7} = \dfrac{-7}{-7} = 1$

(e) $\cos(0) = 1$ (f) $|-7| = 7$

(g) $-7+0 = -7$ (h) $1/-7 = -1/7$

#7) (a) all x: $(-\infty, \infty)$ (b) all $x \geq 0$: $[0, \infty)$
(c) all $x \neq 0$: $(-\infty, 0) \cup (0, \infty)$ (d) all $x > 0$: $(0, \infty)$

Challenge

#16), p. 145 $f(x) = \begin{cases} 1/n & \text{if } x = m/n \text{, lowest terms (rational)} \\ 0 & \text{if irrational} \end{cases}$

(a) If a is rational, with $a = m/n$ in lowest terms, then $f(a) = 1/n$. Using $\varepsilon = (1/n) \cdot \frac{1}{2}$ we have $|f(x) - f(a)| = \frac{1}{n} > \varepsilon$ for all x irrational.

Since every δ interval around a has irrational x in it, this is discontinuous at all rationals.

(b) If $\varepsilon > 0$ is given and If a is irrational, then look at $f(x) \geq \varepsilon > 0$: there are finitely many x such that $f(x) \geq \varepsilon$ since once $\frac{1}{n} < \varepsilon$ the rationals m/n in lowest terms have $f(x) < \varepsilon$. But we can pick $\delta = \min\{|a - r_1|, |a - r_2|, \ldots, |a - r_N|\}$ where $r_1, \ldots, r_N$ are all the rationals with $f(r) \geq \varepsilon$. So for any $\varepsilon > 0$, we can find δ so that $|x - a| < \delta$ implies $|f(x) - f(a)| = |f(x) - 0|$
$= f(x) < \varepsilon$

(c) Rough sketch [with vertical segments of "ruler" too]