MATH 113 - Fall 2004 - Prof SACHS

HOMEWORK 2  COMMENTS / SOLUTIONS

#5 (a) Using Q₁ = (10, 120), Q₂ = (14, 380), Q₃ = (16, 470), Q₄ = (18, 530)

Slopes are: P₁Q₁ = \(\frac{430}{10} = 43\), P₂Q₂ = \(\frac{270}{6} = 45\), P₃Q₃ = \(\frac{180}{4} = 45\), P₄Q₄ = \(\frac{10}{2} = 50\)

(b) ESTIMATE 50 as slightly higher.

#10 (a) Limit is 0 [\"fill in value\"]  (b) Limit is -1  (c) Doesn't exist - can't match right/left

#12 (a) False - limit is 1  (b) False - wrong value  (c) True - mismatch L/R

(d) True - values are on graph  (e) True - limit exists, even at 2

#32 \(\left|2x - 2\right| - \left(-6\right)\) < 0.02 - Simplify inside abs. value:

\[2x - 2 + 6 = 2x + 4 = 2(x + 2)\] so \(\left|2(x + 2)\right| = 2|x + 2|\)

We find \(\frac{2}{|x + 2|} < 0.02\) when \(|x + 2| < 0.01\) [divide by 2]

Thus \(-2.01 < x < -2 + 0.01 = -1.99\) and \(\delta = 0.01\) works.

#34 \(\left|\sqrt{19 - x} - 3\right| < 1 \iff -1 < \sqrt{19 - x} - 3 < 1\) which

is equivalent to: \(2 < \sqrt{19 - x} < 4\)

Therefore \(4 < 19 - x < 16\) - solve for \(x\) in parts

\(4 < 19 - x \iff x < 19 - 4 = 15\) and \(19 - x < 16 \iff 3 < x\)

So \(3 < x < 15\) - with back 10 we can go 7 left, 5 right

So \(\delta = 5\) or smaller works.

#12 (a) \(\lim_{x \to -2} \left(r^3 - 2r^2 + 4r + 8\right) = -8 - 8 - 8 + 8 = -16\)

(b) \(\lim_{x \to -2} \frac{x + 3}{x + 6} = \frac{2 + 3}{2 + 6} = \frac{5}{8}\)
(c) \( \lim_{y \to -3} \sqrt[3]{(5-y)} = 8 \quad \sqrt[3]{2} = 2^4 = 16 \)

(d) \( \lim_{\theta \to 5} \frac{\theta - 5}{\theta^2 - 25} \quad : \quad 0 \) unless we divide/factor

\[= \lim_{\theta \to 5} \frac{\theta - 5}{(\theta - 5)(\theta + 5)} = \lim_{\theta \to 5} \frac{1}{\theta + 5} = \frac{1}{5 + 5} = \frac{1}{10} \]

22. (a) \( \lim_{x \to 2^+} f(x) = 2 \) from graph; \( \lim_{x \to 2^-} f(x) = 1 \) also

(b) It doesn't exist: L/R mismatch

(c) \( \lim_{x \to 4^+} f(x) = 3 \) \( \lim_{x \to 4^-} f(x) = 3 \) - limit exists, equals 3.

24. (a) \( \lim_{x \to 0^+} g(x) = 0 \) by sandwich; \( -\sqrt{x} \leq g(x) \leq \sqrt{x} \)

(b) \( \lim_{x \to 0^-} g(x) \) doesn't exist since domain is \( x > 0 \).

(c) No - no limit from left implies no limit.

36. (a) \( \lim_{x \to 2} \frac{x - 5}{x - 2} = 3 \) - since denominator \( \to 0 \), most \( \lim_{x \to 2} f(x) = 5 \)

(b) Similarly, \( \lim_{x \to 2} f(x) = 5 \) again.