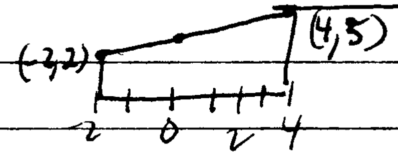
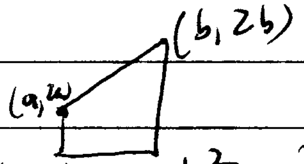
MATH 113 – Prof. SACHS – SOLUTION TO HOMEWORK 10

$\boxed{4.4}$ #17 $\int_{-2}^{4}\left(\frac{x}{2}+3\right)dx$



= area of trapezoid = (avg of lengths at ends) × base $=\left(\frac{2+5}{2}\right)\cdot 6 = 21$

#22 $\int_a^b 2s\,ds \quad 0<a<b$



$=\frac{1}{2}(2b+2a)(b-a) = (b+a)(b-a) = b^2-a^2$

#37 $\int_0^1 \frac{1}{1+x^2}\,dx$; $f(0)=1$, $f(1)=\frac{1}{1+1}=\frac{1}{2}$, $\frac{1}{2}\le\frac{1}{1+x^2}\le 1$

so $\frac{1}{2}\le\int_0^1\frac{1}{1+x^2}\,dx \le 1$

#38 at $x=\frac{1}{2}$, $\frac{1}{1+1/4}=\frac{1}{5/4}=4/5$ → get $\frac{4}{5}\cdot\frac{1}{2}\le\int_0^{0.5}\frac{1}{1+x^2}\,dx\le 1\cdot\frac{1}{2}$

and $\frac{1}{2}\cdot\frac{1}{2}\le\int_{0.5}^{1}\frac{1}{1+x^2}\,dx \le \frac{4}{5}\cdot\frac{1}{2}$ → get $\underbrace{\frac{4}{5}\cdot\frac{1}{2}+\frac{1}{2}\cdot\frac{1}{2}}_{=\,.65}\le\int_0^1\frac{1}{1+x^2}\,dx\le\frac{1}{2}+\frac{4}{5}\cdot\frac{1}{2}$

$=\frac{9}{10}$

$\boxed{4.5}$ #10 $\int_{-1}^{1}(r+1)^2\,dr$ could do using chain rule (a) or expanded (b)

(a) $=\frac{(r+1)^3}{3}\Big]_{-1}^{1}=\frac{2^3}{3}-\frac{0^3}{3}=\boxed{8/3}$

(b) $\int_{-1}^{1}(r+1)^2\,dr=\int_{-1}^{1}(r^2+2r+1)\,dr$

$=r^3/3+r^2+r\Big]_{-1}^{1}=\frac{1}{3}+1+1$

$-\left(-\frac{1}{3}+1-1\right)$

$=\boxed{8/3}$

#11 $\int_1^{\sqrt{2}}\left(\frac{u^2}{2}-\frac{1}{u^5}\right)du=\frac{u^3}{6}+\frac{1}{4}u^{-4}\Big]_1^{\sqrt{2}}=\frac{(\sqrt{2})^3}{6}+\frac{1}{4}(\sqrt{2})^{-4}-\frac{1}{6}-\frac{1}{4}$

#17) $\frac{d}{dt}\int_0^{t^4}\sqrt{u}\,du$ (a) $\int_0^{t^4}\sqrt{u}\,du = \frac{u^{3/2}}{3/2}\Big]_0^{t^4} = \frac{(t^4)^{3/2}}{3/2} - 0 = \frac{2t^6}{3}$

so $\frac{d}{dt}\int_0^{t^4}\sqrt{u}\,du = \frac{d}{dt}\left(\frac{2t^6}{3}\right) = \frac{2}{3}\cdot 6t^5 = 4t^5$ (b) $\frac{d}{dt}\int_0^{t^4}\sqrt{u}\,du$

let $t^4 = s$ then $\frac{d}{dt}\int_1^{t^4}\sqrt{u}\,du = \left(\frac{d}{ds}\int_0^{s}\sqrt{u}\,du\right)\frac{ds}{dt}$. (chain rule)

$= \sqrt{s}\cdot 4t^3 = \sqrt{t^4}\cdot 4t^3 = t^2\cdot 4t^3 = 4t^5$ ✓

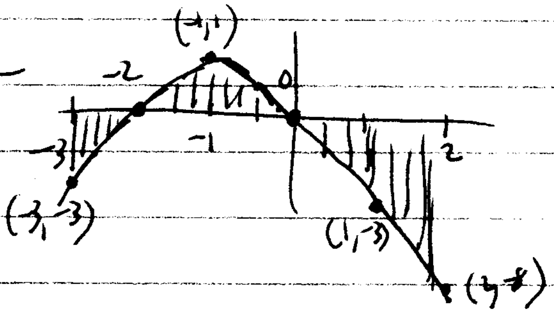
#20) $y = \int_1^x \frac{1}{t}\,dt,\ x>0 \rightarrow \frac{dy}{dx} = \frac{1}{x}$ using FTC

#26) $\int_0^1 t\sqrt{t^2+1}\,dt$ let $u = t^2+1$, $du = 2t\,dt$ $> \int \frac{1}{2}u^{1/2}\,du$

get antiderivative

$= \frac{1}{2}\cdot\frac{3}{2}\cdot u^{3/2} \rightarrow \frac{3}{4}(t^2+1)^{3/2}\Big]_0^1 = \frac{3}{4}(2^{3/2}) - \frac{3}{4}$

#33) $y = -x^2-2x,\ -3 \le x \le 2$



Area has 3 pieces — want to use positive terms in each!

$$\int_{-3}^{-2}\left[0-(-x^2-2x)\right]dx + \int_{-2}^{0}(-x^2-2x)\,dx + \int_0^2\left[0-(-x^2-2x)\right]dx$$

top − bottom

$$=\int_{-3}^{-2}(x^2+2x)\,dx + \int_{-2}^{0}(-x^2-2x)\,dx + \int_0^2(x^2+2x)\,dx$$

$$= \frac{1}{3}x^3+x^2\Big]_{-3}^{-2} + \left(-\frac{1}{3}x^3-x^2\right)\Big]_{-2}^{0} + \frac{1}{3}x^3+x^2\Big]_0^2$$

$$= \frac{1}{3}(-8)+4-\frac{1}{3}(-27)-9+0+\frac{1}{3}(-8)+4+\frac{8}{3}+4-0 = \frac{28}{3}$$