WRITING ABOUT MATHEMATICS: PROOFS, IMAGES, AND "PLAIN WORDS"

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There are many goals that we professors want students to attain over a semester. Some are "material specific", such as familiarity with a specific body of knowledge, or facility with specific techniques to solve problems. But there are also themes that run across disciplines and which are valued in the college-educated world at large. These include the ability to write well, think logically, and present a story. In mathematics we often channel these last goals into "proof writing", or writing logically sound arguments to "prove" a mathematical statement. But writing about mathematics and science is not limited to proofs; compelling and explanatory arguments about mathematical topics may also include pictures, graphics, description, and supporting discussion. In MATH 321 (Abstract Algebra), I designed two assignments to help students do the latter.

First, let me provide some background on the math involved. One of the major topics in Abstract Algebra is "groups". There are many different groups, each with different properties. The study of groups has had fundamental and profound implications for mathematics. Its application to and influence on fields as diverse as computer science, cryptography, information security, crystallography, and quantum mechanics cannot be overstated. Groups can also be used to study the symmetry of common place objects or art. For example, a square has symmetry: if you rotate it 90 degrees, you get back the same object. On the other hand, if you rotate the square 45 degrees, you can tell a difference from the "original" square, because it will "look" rotated. So a "rotation by 90 degrees" is a *symmetry* of the square. The set of all symmetries of an object forms a group.

In each of two assignments I designed (which grew out of a Technology Across the Curriculum image manipulation workshop), students were asked to download three images from an on-line "library" that I created for the course. The library was culled from many web pages; some images are actual photographs (such as a Mosaic from Spain, or a rug from Iran) and others were computer generated drawings (such as a virtual snowflake or the "flower" below).

The students were then asked to compose a coherent essay, including the pictures, to describe the symmetries of these objects using mathematical language. The students were asked to use some photo-manipulation and/or drawing tools to illustrate their arguments. *Proof* was not the criterion for a good paper. Good presentation, correct mathematics, and a clear, convincing and thorough argument were the basis on which I graded these papers.

Overall I felt the writing-technology assignments were successful in teaching the students a different way of using their mathematical background towards careers that may not directly include traditional proof. The students' responses to the first assignment included incomplete sentences, paragraphs formatted with great gaps in which pictures would fill in a small space, and random incoherent thoughts on symmetry. We had interesting classroom discussions about what communication is, and how to communicate effectively (as well as how to understand the symmetry of two-dimensional patterns!), and how to resize pictures or wrap text around them. Weren't these students already supposed to know this? Apparently not, and it was my responsibility to teach them what they were supposed to do. I was impressed and buoyed by the second set of essays, even as the mathematics was more challenging. Students were precise and descriptive. They spent much more time than they might have otherwise thinking about symmetry, for writing about something is much more challenging than convincing yourself you understand it for a midterm exam.

Surprisingly, these writing assignments took away very little time from the "traditional" proof-writing, perhaps because problem sets and essays are so different by nature. The students themselves felt proud of their work in a way they never expressed about the proofs they wrote. As a professor, I accomplished the unimaginable: the students worked harder and learned more.