**Math 351, Probability**
**Problem Set # 8**
**Due Thursday, November 29**

1. A pair of fair dice, one red and one blue, are rolled until at least one of them shows a 1. Let $X$ be the number of times the red die is rolled and let $Y$ be the number of times the blue die is rolled.

   (a) Find the joint pmf of $X$ and $Y$. (Hint: The two dice are rolled the same number of times.)
   (b) Find the marginal pmf of $Y$.
   (c) Find $E(X + 2Y)$.

2. The joint probability density function of $X$ and $Y$ is given by $f(x, y) = 2$, for $0 < x < yx < 1$.

   (a) Sketch the region on which $f(x, y) > 0$.
   (b) Verify that $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y)dydx = 1$.
   (c) Find the marginal density functions $f_X(x)$ and $f_Y(y)$.
   (d) Are $X$ and $Y$ independent? Why or why not?

3. The joint probability density function of $X$ and $Y$ is given by $f(x, y) = xy$, for $0 < x < 1$, $0 < y < 2$.

   (a) Find the marginal densities of $X$ and $Y$.
   (b) Find $P\{X < Y\}$.
   (c) Find the density function of $Z = X + 2Y$.

4. $X$ and $Y$ are independent uniform $(0, 1)$ random variables. Find the density of $min(X, Y)$.

5. Suppose that $X$ and $Y$ are independent normal random variables, with $\mu_X = 5$, $\sigma_X^2 = 9$ and $\mu_Y = -1$, $\sigma_Y^2 = 4$.

   (a) Find $P\{X > Y\}$.
   (b) Find $P\{X + Y > 3\}$.

6. Suppose that $X$ and $Y$ are independent exponential random variables, with $\lambda_X = 2$ and $\lambda_Y = 2$. Find the density of $Z = X + Y$.

7. If $X, Y, Z$ are independent exponential random variables, each with parameter $\lambda = 1$, find the probability that the largest of the three is greater than the sum of the other two.