MATH 351
Problem Set 7
Due November 15, 2012

1. Suppose that X is a normal random variable with parameters $\mu = 1$ and $\sigma^2 = 9$.

   (a) Find $P\{-2 \leq X \leq 1\}$

   (b) Find $E[X]$ and $Var(X)$.

   (c) What is the distribution of $Y = 2X - 1$? In other words, what kind of random variable is it, and what are its parameters?

2. Suppose that the heights of the men of a certain large city are normally distributed with mean 71 inches and variance 15 inches. Find the probability that a randomly chosen man is at least 5 feet tall and no more than 6'1" feet tall. What is the conditional probability that the randomly chosen man is at least 6 foot six, given that he is at least 6'1" feet tall?

3. What is the probability that, of the first four men we meet from the city of problem 2, exactly four are at least 6'1" feet tall? What assumptions are you making?

4. Suppose that a fair coin is flipped 100 times. Find the probability that exactly 50 of the tosses are heads, using both the exact probability given by the binomial distribution and the approximation given by the normal distribution.

5. Suppose that 15% of the student population at small colleges consists of Gary Johnson fans (Libertarian candidate for president in most states). If a college has 300 students, what is the probability that at least 35 of them are Gary Johnson fans? What assumptions are you making?

6. Suppose that X is exponentially distributed with $\lambda = 3$.

   (a) What is $P\{X > 2\}$?

   (b) What is $P\{X > 5|X > 3\}$?

   What did you notice about these two answers? Is it a coincidence?

7. Suppose X is exponentially distributed with $\lambda = 3$. Find a number m for which $P\{X \leq m\} = 0.5$. Find $\mu = E[X]$. Are μ and m the same?

8. Evaluate the integral
$$\int_0^\infty x^7 e^{-2x}\,dx.$$