Mathematics 108, Introductory Calculus
Test 1, Sections 1.4-1.6, questions from old tests

1. Consider the function: \( f(x) = \frac{x^2 + x - 6}{2x^2 - 6x + 4} \). Find each of the following limits, if the limit exists. If the limit does not exist, write DNE.

a) \( \lim_{x \to 2} f(x) = \lim_{x \to 2} \frac{(x+3)(x-2)}{(x+2)(x-3)} = \lim_{x \to 2} \frac{x+3}{x+2} = \frac{5}{2} \)

b) \( \lim_{x \to 1} f(x) = \text{DNE (vertical asymptote)} \)

c) \( \lim_{x \to 3} f(x) = \lim_{x \to 3} \frac{9-3-6}{2(9)-18+4} = \lim_{x \to 3} \frac{0}{0} = \text{DNE} \)

d) \( \lim_{x \to 0} f(x) = \lim_{x \to 0} \frac{x^2 + x - 6}{x^2 - 6x + 4} = \lim_{x \to 0} \frac{1+\frac{1}{x} - \frac{6}{x^2}}{2 - \frac{6}{x} + \frac{4}{x^2}} = \frac{1}{2} \)

2. Find the following limits, if the limits exist. If the limit does not exist, write DNE.

a) \( \lim_{x \to \infty} (x^2 + 2x + 7) = +\infty \) (limit DNE: quadratic function, grows without bounds)

b) \( \lim_{x \to \infty} \frac{3-x^2}{4x^2 + 5} = \lim_{x \to \infty} \frac{3 - \frac{x^2}{x^2}}{4 + \frac{5}{x^2}} = \lim_{x \to \infty} \frac{3}{4} = \frac{3}{4} \)

c) \( \lim_{x \to 0} \frac{3x^2 - 7}{x^4 - 1} = \lim_{x \to 0} \frac{3x^2 - \frac{7}{x^2}}{x^4 - \frac{1}{x^4}} = \lim_{x \to 0} \frac{0}{0} = 0 = 0 \)

3. Given the graph of \( g(x) \), find (if they exist)

a) \( \lim_{x \to 0^+} g(x) = \frac{3}{4} \)

b) \( \lim_{x \to 0^-} g(x) = \frac{3}{4} \)

c) \( \lim_{x \to 0} g(x) = \text{DNE (because } \lim_{x \to 0^-} g(x) \neq \lim_{x \to 0^+} g(x) \)\)

d) \( g(0) = 2 \)

e) Is \( g(x) \) continuous at \( x = 0 \)? \( \text{No} \) Briefly explain why or why not, using mathematical concepts in your answer.

\( g(x) \) is not continuous at \( x = 0 \) because \( \lim_{x \to 0^-} g(x) \) does not exist. \( \text{Note: it is continuous everywhere else!} \)
4. Is the function \( f(x) \) below continuous at \( x = 3 \)? \[ \text{Yes} \] Justify your answer mathematically (not just with a graph).

\[
f(x) = \begin{cases} 
  x^2 + 3, & \text{if } x \leq 3 \\
  4x, & \text{if } x > 3
\end{cases}
\]

\[
\lim_{x \to 3^-} f(x) = \lim_{x \to 3} x^2 + 3 = 3^2 + 3 = 12
\]

\[
\lim_{x \to 3^+} f(x) = \lim_{x \to 3} 4x = 4 \cdot 3 = 12
\]

\[
\lim_{x \to 3} f(x) = \lim_{x \to 3} x^2 + 3 = \lim_{x \to 3} 4x = 12
\]

5. List all values, if any, at which the following functions are discontinuous. Describe any discontinuities you find.

a) \( f(x) = x^3 - 15x^4 + 9x - 11 \)

No discontinuities (polynomials are continuous everywhere)

b) \( g(x) = \frac{x}{2x^2 - 8x} \)

Discontinuities at \( x = 0, x = 4 \)

6. A bookstore has been offering a special commemorative book at a price of $15 per book, and at that price, has been selling 24 books per month. The bookstore is planning to reduce the price to stimulate sales and estimates that for each $1 reduction in price, 8 more books will be sold each month.

a) Find the linear demand function that models the facts above. Express the demand \((D(p))\) for the book as a function of the price \(p\) at which the book is sold.

\[
\frac{\Delta D}{\Delta p} = \frac{48 \text{ (more books)}}{1 \text{ (less cost)}}
\]

\[
D = 24 - 8(p - 15) = -8p + 120
\]

\[
D = -8 + 144 = -8p + 144
\]

b) Express the total revenue which the bookstore will receive as a result of the sales of the commemorative book as a function of the price \(p\) of the books.

\[
R(p) = p \cdot D(p) = p(-8p + 144) = -8p^2 + 144p
\]

The bookstore can obtain the book from the publisher at a cost of $6 per book.

a) Express the total profit which the bookstore can make on the sale of the books as a function of the price \(p\) of the books.

\[
\text{Profit} = R(p) - C(p) = -8p^2 + 144p - 6(-8p + 144) = -8p^2 + 192p - 6 \cdot 144
\]

b) At what price should the books be sold to generate the greatest profit? What will the store's total profit be at that price?

\[
p = \frac{-b}{2a} = \frac{-192}{2(-8)} = \frac{-192}{-16} = 12
\]

\[
P(12) = -8(144 - 24(12) + 108) = -8(-36) = 288
\]

c) How do you know that profit is maximized at that point? Because \( P(p) = -8p^2 + 112p - 864 \) is a parabola opening down \((a = -8 < 0)\), so the vertex is a maximum. Coordinates of the vertex are \((12, 288)\).