Math 108
Extra credit problem: 3/6/09

Let \( f(x) = 2x^2 - 6x \)

1. Find \( f'(x) \) using the limit definition.
   \[
   f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{2(x+h)^2 - 6(x+h) - (2x^2 - 6x)}{h}
   \]
   \[
   = \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) - 6x - 6h - 2x^2}{h} = \lim_{h \to 0} \frac{4x + 4h - 6}{h} = 4x - 6
   \]

2. Find slope of line tangent to \( f(x) \) at \( x=1 \).
   Slope = \( f'(1) = 4 \times 1 - 6 = -2 \) = \text{mean}

3. Find equation of the line tangent to \( f(x) \) at \( x=1 \).
   Point: find \( f(1) = 2(1)^2 - 6(1) = 2 - 6 = -4 \). Point is \((1, -4)\)
   (both the function and tangent line go through this point)
   Equation: \( y - (-4) = -2(x - 1) \)
   \[
   \begin{align*}
   y + 4 & = -2x + 2 \\
   \text{or} & \\
   y & = -2x - 2
   \end{align*}
   \]

4. Graph \( f(x) \) and line tangent at \( x=1 \).
   \( f(x) = 2x^2 - 6x = 2x(x - 3) \)
   \( x \)-intercepts at \( x = 0, x = 3 \)
   \( x \)-coord of vertex at \( \frac{-f'(x)}{2} = \frac{1}{4} = 1/2 \)
   \( y \)-coord of vertex: \( 2(1/2)^2 - 6(1/2) = 2 \times \frac{9}{4} = 9 \)
   \( y - \frac{0}{2} + \frac{-9}{2} = -4.5 \)