Math 108, Business Calculus
Quiz, Implicit Differentiation

April 10, 2009

Please show all work neatly. Use of calculators is not permitted.

1. Find the derivative $y'$ or $\frac{dy}{dx}$, where $xy + 5y = x^4$. You may use implicit differentiation or you may solve by differentiating an explicit formula for $y$.

\[ y + xy' + 5y = 4\sqrt{3} \]
\[ xy + 5y = 4x^3 - y \]
\[ y'(x+5) = 4x^3 - y \]
\[ y' = \frac{4x^3 - y}{x+5} \]

or:
\[ y = \frac{x^4}{x+5} \]
\[ f = x^4 \quad g = x+5 \]
\[ f' = 4x^3 \quad g' = 1 \]
\[ \frac{4x^3 - 4x^4}{(x+5)^2} \]

2. Suppose that $x^2(x+y)^5 + y^5 = 3$.

a) Find the derivative $y'$ or $\frac{dy}{dx}$ using the equation above. You must use implicit differentiation to solve this problem (doing algebra first is always allowed.)

\[ x^3 + 3x^2y + y^5 = 3 \]
\[ 3x^2 + 2xy + 2y' + 5y^4 \cdot y' = 0 \]
\[ -3x^2 - 2xy \]
\[ x^2y' + 5y^4 \cdot y' = -3x^2 - 2xy \]
\[ y' = \frac{-3x^2 - 2xy}{x^2 + 5y^4} \]

b) Find an equation of the line tangent to the curve above at the point $(1, 1)$.

\[ m = \frac{-3(1)^2 - 2(1)(1)}{1^2 + 5(1)^4} = \frac{-3 - 2}{1 + 5} = -\frac{5}{6} \]

Equation:
\[ y - 1 = -\frac{5}{6}(x - 1) \]
\[ y = -\frac{5}{6}x + \frac{11}{6} \]
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Please show all work neatly. Use of calculators is not permitted.

1. Find the derivative $y'$ or $\frac{dy}{dx}$, where $xy + 4y = x^3$. You may use implicit differentiation or you may solve by differentiating an explicit formula for $y$.

\[
y + xy' + 4y' = 3x^2
\]
\[
xy' + 4y' = 3x^2 - y = y'(x + 4)
\]
\[
y' = \frac{3x^2 - y}{x + 4}
\]
\[
y' = \frac{y(x + 4) = x^3}{x + 4}
\]
\[f(x) = x^3, \quad g(x) = x + 4
\]
\[f'(x) = 3x^2, \quad g'(x) = 1
\]
\[y' = \frac{3x^2(x + 4) - x^3}{(x + 4)^2}
\]

2. Suppose that $x^3(x + y) + y^4 = 3$.
   a) Find the derivative $y'$ or $\frac{dy}{dx}$ using the equation above. You must use implicit differentiation to solve this problem (doing algebra first is always allowed).

\[
x^4 + x^3y + y^4 = 3
\]
\[
4x^3 + 3x^2y + x^3y' + 4y^3y' = 0
\]
\[
x^3y' + 4y^3y' = -4x^3 - 3x^3y
\]
\[
y' = \frac{-4x^3 - 3x^3y}{x^3 + 4y^3}
\]

b) Find an equation of the line tangent to the curve above at the point $(1, 1)$.

At $(1, 1)$, $m_{\text{tan}} = \frac{-4(1) - 3(1)}{1 + 4} = \frac{-7}{5}$

Equation: 
\[y - 1 = \frac{-7}{5}(x - 1)\] or 
\[y = -\frac{7}{5}x + \frac{12}{5}\]