1. Use the limit definition of the derivative, that is \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \), to find the derivative of \( f(x) = 5x - 3x^2 \). You may check your answer with the power rule, but you must show all steps of your reasoning to get any credit for this answer.

\[
f'(x) = \lim_{h \to 0} \frac{5(x+h) - 3(x+h)^2 - (5x - 3x^2)}{h} = \lim_{h \to 0} \frac{5h - 6xh - 3h^2}{h} = \lim_{h \to 0} \frac{5h}{h} - \frac{6xh}{h} - \frac{3h^2}{h} = 5 - 6x
\]

2. Find the following derivatives, using any correct techniques. Unless otherwise specified, you do not have to simplify your answers.

a) \( f(x) = \frac{4x-3}{x^2+4} \)

\[
f'(x) = \frac{(x^2+4)(4) - (4x-3)(2x)}{(x^2+4)^2} = \frac{4x^2 + 16 - 8x^2 + 6x}{(x^2+4)^2} = \frac{4 - 4x^2 + 6x}{(x^2+4)^2}
\]

b) \( f(x) = (x^2 - 2\ln(x))^{1/2} \)

\[
f'(x) = \frac{1}{2}(x^2 - 2\ln(x))^{-1/2} \cdot (2x - 2/x)
\]

c) \( f(x) = \sqrt[3]{3x^3 - 4} \)

\[
f'(x) = \frac{1}{3}(3x^3 - 4)^{-2/3} \cdot 9x^2 = \frac{9x^2}{3(x^3 - 4)^{2/3}}
\]

3. Find the first and second derivatives of each of the following:

a) \( f(x) = 2x^2 + \frac{3}{x} \)

\[
f'(x) = 4x - 7x^{-2}
\]
\[
f''(x) = 4 + 14x^{-3}
\]

b) \( f(x) = (x^4 - 2)^6 \)

\[
f'(x) = 6(x^4 - 2)^5(4x^3)
\]
\[
f''(x) = 24x^3(20x^3)(x^4 - 2)^4 + 72x^2(x^4 - 2)^5
\]
4. Find an equation of the line tangent to \( f(x) = x^3(2 - 3x^2) \) at \( x = 1 \).

**Solution:**

I. Point: plug \( x = 1 \) into \( f(x) \) to get \( y: f'(1) = 13(2 - 3, 12) = -1 \). \( (1, -1) \) is the point.

II. Slope: \( f'(x) = 3x^2(2 - 3x^2) + (-9x^4)x^2 \) (product rule or -

\[
\begin{align*}
&f'(x) = 3x^2 (2 - 3x^2) - 9x^4, \\
&f'(1) = 3x^2 - 9x^4, \quad f'(1) = 3(x^2 - 3x^4) = -9x^2.
\end{align*}
\]

III. Line: \( y - (-1) = -12(x - 1) \) \( y + 1 = -12x + 12 \); \( y = -12x + 11 \).

5. An upscale fast food restaurant has determined that the relationship between the price \( p \), in dollars, at which it can sell hamburgers and the quantity \( q \) that it can sell is \( p = \frac{80,000 - q}{20,000} \).

a) Find the revenue function as a function of the quantity of hamburgers sold (ie, find \( R(q) \)).

\[
R(q) = q \cdot \left( \frac{80,000 - q}{20,000} \right) = \frac{80,000q - q^2}{20,000} = \frac{1}{20,000} \left( 80,000q - q^2 \right).
\]

b) Find the marginal revenue function.

\[
R'(q) = \frac{1}{20,000} \cdot (80,000 - 2q) = \frac{80,000 - 2q}{20,000}
\]

c) Use the marginal revenue function to estimate the revenue from the sale of the 10,001\(^{st}\) hamburger and interpret your result (tell me in words, with units, what your answer means.)

\[
R'(10,000) = \frac{1}{20,000} \left( 80,000 - 2(10,000) \right) = \frac{1}{20,000} \left( 80,000 - 20,000 \right) = \frac{60,000}{20,000} = 3 \text{ dollars per hamburger}
\]

6. When a company produces and sells \( x \) thousand units per week, its total weekly profit is \( P \) thousand dollars, where \( P = \frac{300x}{200 + x^2} \).

The production level, in thousands of units, at \( t \) weeks from the present is \( x = 2 + 3t \).

Find the function that models how fast profits are changing with respect to time (that is, find \( \frac{dP}{dt} \)). You do not need to simplify your answer.

\[
P(t) = \frac{300(2+3t)}{200 + (2+3t)^2} = \frac{600 + 900t}{200 + 4 + 12t + 9t^2} = \frac{600 + 900t}{204 + 12t + 9t^2}
\]

\[
P'(t) = \frac{900(204 + 12t + 9t^2) - (12 + 18t)(600 + 900t)}{(204 + 12t + 9t^2)^2}
\]

(Extra credit: What happens to profits in the long run? How do you know?)

If \( t \) is asking \( \lim_{t \to \infty} \frac{600 + 900t}{204 + 12t + 9t^2} = 0 \) (profits approach \( 0 \) as \( t \to \infty \) (power in numerator is larger than power in denominator).