1. Sketch the graph of the polynomial \( P(x) = x^4 - 3x^3 + 2x^2 \). Be sure that your graph shows all intercepts and exhibits the proper end behavior.

\[
P(x) = x^4 - 3x^3 + 2x^2
\]

- Zeros at \( x = 0 \) (double root)
- \( x - 2 = 0 \) \( \Rightarrow x = 2 \)
- \( x - 1 = 0 \) \( \Rightarrow x = 1 \)

\[
P(-1) = (-1)^4 - 3(-1)^3 + 2(-1)^2 = 1 + 3 + 2 = 6
\]

\[
P('1/2') = (1/2)^4 - 3(1/2)^3 + 2(1/2)^2 = \frac{1}{16} - \frac{3}{8} + \frac{2}{4}
\]

\[
P(3/2) = (3/2)^4 - 3(3/2)^3 + 2(3/2)^2 = \frac{81}{16} - \frac{3\cdot27}{8} + 2\left(\frac{9}{4}\right)
\]

2. True or False: The graph below could be the graph of the function \( P(x) = x^6 - 4x^4 \).

Give at least two good reasons to justify your answer, including information on x- and y-intercepts, multiplicity of roots and end behavior of the function.

**True**

- Degree of \( P(x) \) is even
- Degree of polynomial in graph is odd
- Leading coefficient of \( P(x) \) is positive, polynomial in graph is reflected across x-axis, so its leading coefficient is negative.
- Multiplicity of zero at \( x = 0 \) is even, so graph should "bounce" there, but graph crosses x-axis. 

**False**

- To find y-intercept, match \( P(0) = 0 \) 
- \( P(x) = x^2(x-2) \)
- Intercepts at \( x = 0, x = 2, x = 2 \), as in the graph.

2 points for each justification

(ned to have 2 if they have 4)
Math 105, Precalculus  
Quiz 6, Section 3.1  
November 5, 2009

Show all work neatly and completely. Use of calculators is not permitted.

1. Sketch the graph of the polynomial \( P(x) = -x^3 + x^2 + 12x \). Be sure that your graph shows all intercepts and exhibits the proper end behavior.

\[
P(x) = -x^3 + x^2 + 12x
\]

\[
= -x(x-4)(x+3)
\]

Let \( x = 0 \): \( x = 0 \)

Let \( x = 4 \): \( x = 4 \)

Let \( x = -3 \): \( x = -3 \)

\( \text{zeros (all of multiplicity 1)} \)

\[
P(1) = -(1+1+12) = -14
\]

\[
P(-1) = -(-1)^3 +(-1)^2 +12(-1) = 1 -1 -12 = -10
\]

\[
P(2) = -8 + 4 + 24 = 20
\]

2. True or False: The graph below could be the graph of the function \( P(x) = 5x^4 - 10x^3 \).

\[
P(x) = 5x^4 - 10x^3
\]

\[
= 5x^3(x-2)
\]

\( \text{zeros at } x=0, x=2.\)

(consistent with graph)

Give at least two good reasons to justify your answer, including information on \( x \)- and \( y \)-intercepts, multiplicity of roots and end behavior of the function.

I. Degree of \( P(x) \) is even, but the degree of the polynomial in the graph is odd.

II. Leading coefficient of \( P(x) \) is positive, but the polynomial in the graph is reflected across the \( x \)-axis, so its leading coefficient is negative.

III. Multiplicity of the zero at \( x=0 \) is even (function "bounces" there) but multiplicity of zero at \( x=0 \) in \( P(x) \) is 1 (graph must cross \( x \)-axis.)