MATH 105  Homework 3  ANSWER KEY

1. Schum 3.2.

\[ \frac{2x^3 + 4x^2 + 8}{x^2 + 0x - 2} \]

\[ \frac{2x^5 + 4x^4 - 4x^3 + 10x^2 - x - 3}{2x^5 + 0x^4 - 4x^3} \]

\[ = \frac{4x^4 + 0 + 0}{-4x^4 + 0 + 8x^2} \]

\[ = \frac{8x^2 - x - 3}{-8x^2 + 0 - 16} \]

\[ = \frac{-x + 13}{-x + 13} \]

\[ \therefore 2x^5 + 4x^4 - 4x^3 - x - 3 = (x^2 - 2)(2x^3 + 4x^2 + 8) + (-x + 13) \]

(Note: this one can not be done with synthetic division)

18.

\[ \frac{3x^2 - 8x - 1}{x^2 + x + 3} \]

\[ \frac{3x^4 - 5x^3 + 0x^2 - 20x - 5}{3x^4 + 3x^3 + 9x^2} \]

\[ = \frac{-8x^3 - 9x^2 - 20x}{-8x^3 - 8x^2 - 24x} \]

\[ = \frac{-x^3 + 4x - 5}{-x^3 - x - 3} \]

\[ \therefore \frac{3x^4 - 5x^3 - 20x - 5}{x^2 + x + 3} = \left( \frac{3x^2 - 8x - 1}{x^2 + x + 3} \right) + \frac{5x - 2}{Q(x)} \]

\[ \text{Q(x) + R(x) D(x)} \]

(0 x^2 inserted to keep columns straight)

On these 2 problems, just take 3/8 1/2 point for each error, up to 2 pts.
for the entire problem.

If they solve correctly, with no "place-holder" zeros, don't take
anything off. But if place holders
get them in trouble, take off 1 point.

Don't take off points if answers
aren't exactly in the form on this
sheet (the from the book zero for),
but you may add 1/2 pt per problem
for getting the form exactly right.
2. \( P(x) = 6x^4 - 7x^3 - 12x^2 + 3x + 2, \quad c = -1, \quad c = 2 \)

a) Show \( c = -1 \) is a zero: A. Plug in: \( P(-1) = 6(-1)^4 - 7(-1)^3 - 12(-1)^2 + 3(-1) + 2 \)

\[ \Rightarrow 6 + 7 - 12 - 3 + 2 = 0 \checkmark \]

or Divide by \( (x+1) \) to check that the remainder = 0.

\[ \frac{6x^2 - 13x + 2}{x+1} \]  

\[ 6x^2 - 7x^2 - 12x^2 + 3x + 2 = (x+1)(6x^3 - 13x^2 + 2) \]

1st for each value (showing it's zero) ordering

Show \( c = 2 \) is a zero: A. Plug in (may plug in to reduced part): \( P(2) = (2+1) \cdot (6 \cdot 2^3 - 13 \cdot 2^2 + 2 + 2) \)

\[ = 3(48 - 52 + 4) = 0 \checkmark \]

or B. Divide \( 2 \) \[
\begin{array}{c|cccc}
6 & 12 & -2 & 2 \\
-12 & -2 & -2 & \\
\hline
6 & 1 & 1 & \\
\end{array}
\]

\( \Rightarrow 0 \in \text{Remainder} = 0; \Rightarrow c = -1 \) is a zero.

(could also have been done with synthetic division.)

b) Zeros at \( x = -1, x = 2 \)

\( 6x^2 - x - 1 = 0 \)

\( (3x + 1)(2x - 1) = 0 \)

\[ \Rightarrow 3x + 1 = 0 \quad 2x - 1 = 0 \]

\[ \Rightarrow x = -\frac{1}{3} \quad x = \frac{1}{2} \]

4 Zeros

(may use quadratic formula, of course)

(consistent with degree of polynomial)
(a) \( P(x) = x^4 - x^3 - 5x^2 + 3x + 6 \); \( c = -1 \), \( c = 2 \)

\[
P(-1) = (-1)^4 - (-1)^3 - 5(-1)^2 + 3(-1) + 6 = 1 + 1 - 5 - 3 + 6 = 0
\]

Divide:

\[
\frac{x^3 - 2x^2 - 3x + 6}{x+1} = x^2 - 3x + 6
\]

\[
(x^4 + x^3)
\]

\[
-2x^2 - 5x^2
\]

\[
-(-2x^2 - 2x^2)
\]

\[
-3x + 3x
\]

\[
-(-3x^3 - 3x)
\]

\[
+6x + 6
\]

\[
\frac{0}{0} \quad \text{(Zero Remainder)}
\]

\( \therefore c = -1 \) is a zero.

Check \( c = 2 \):

\( P(2) = (2+1)(2^3 - 2(2)^2 - 3(2) + 6) = 3(8 - 8 - 6 + 6) = 0 \)

\[
v+1 \\ 2 & -2 & -3 & 6 \\
2 & 0 & -6 \\
1 & 0 & -3 & 0
\]

(b) \( P(x) = (x+1)(x-2)(x^2 - 3) \)

(c) \( \text{Zeros: } x = -1, x = 2, x^2 - 3 = 0 \)

\[
x = \pm \sqrt{3}
\]

3. \( P(y) = y^3 - y^2 - 8y + 12; \ c = 2 \)

\[
y^2 + x - 6
\]

\[
\text{P(2) = 8 - 4 - 16 + 12 = 0} \quad x = 2 \quad y^3 - y^2 - 8y + 12
\]

\[
\text{Divide } \frac{-y^3 + 2y^2}{x^2 - 3x}
\]

\[
-(-x^2 - 2x)
\]

\[
6x + 12
\]

\[
(x - 2)^2(y - 2)
\]

\[
6x + 12
\]

10. c) \( \text{Zeros at } x = 2, x = -3 \) \( \text{(double root at } y = 2) \)
d) Graph:

- $x$-intercepts: $x = 2, x = 3$
- $y$-intercept: $P(0) = 6 + 12 = 18$

Shape: cubic with positive leading coefficient
Zeros: graph crosses $x$-axis at $x = 3$; "bounces" at $x = 2$

Test values:
- $P(4) = -4 - 16 - 144 + 12 = -180 + 14 = -166$
- $P(3) = 27 - 9 - 27 + 12 = 6$

b) $P(x) = x^3 - 7x^2 + 14x - 8$

- Synthetic division: $11 - 7 - 4 - 8$
  - $1 - 6 - 8$

- $1 - 6 - 8 \div 1 = 1$ (is a zero)

- $P(x) = (x - 1)(x^2 - 6x + 8)$
  - $(x - 1)(x - 4)(x + 2)$

- Zeros at $x = 1, x = 4, x = 2$

No points 3b if test values are missing