Honors 225, Homework 4
Name________________________

Problems 1-5 below refer to the Tent map, given by the equations:

\[ T(x) = \begin{cases} 
2x, & x \leq \frac{1}{2} \\
2(1-x), & x \geq \frac{1}{2} 
\end{cases} \text{ on } [0,1] \]

1. Find a point that lies in the subinterval LLR for the tent map \( T \).
   Any \( x \) between \( \frac{1}{8} \) and \( \frac{1}{4} \) will lie in the subinterval LLR. For example, \( x = \frac{1}{8} \).

2. Let \( x_0 \) be a number in the subinterval RLLRRRLLR. Is \( x_0 \) less than, equal to, or greater than \( \frac{1}{2} \)? Same question for \( T^6(x_0) \)?
   \( x_0 \) is \( \bar{\mu} \). \( x_0 = 7/8 \); \( x_6 = T^6(x_0) = \bar{\mu} \). \( T^4(x_0) < \frac{1}{2} \)

3. On a piece of graph paper, carefully draw \( T \), and construct the schematic diagram (similar to the diagram on the bottom of page 28 in the handout) for \( T \), continuing to subintervals corresponding to length 4 sequences.

4. \( T^4 \) has \( 2^4 = 16 \) fixed points. List all itineraries that represent fixed points of \( T^4 \). Which are fixed points of \( T \)? Period-two points? Period-four points? How many periodic orbits of period four are there? Which itineraries are in the same orbit?

5. In this problem, you will set up equations and solve them to find all period-three points of \( T \).
   a) One period-three point \( p \) is represented by the repeating itinerary LLR.
      Set up the equation \( T^3(p) = p \) by composing the appropriate formulas. Solve the equation for \( p \) and give all three points in the orbit.
      \( T(p) = 2p \); \( T(2p) = 4p \); \( T(4p) = 2(1-4p) = 2-8p = p \)
      \( 8p = 2 \Rightarrow p = \frac{2}{8} = \frac{1}{4} \) orbit: \{ \frac{1}{4}, \frac{3}{4}, \frac{2}{4} \}
   b) Another period-three point \( q \) is represented by LRR. Repeat directions for part a) to find all three points in the orbit of \( q \).
      \( T(q) = 2q \); \( T(2q) = 2(1-2q) = 2-4q \); \( T(2-4q) = 2(1-(2-4q)) = 2(-1+4q) = -2+8q = q \Rightarrow q = \frac{2}{7} \) orbit: \{ \frac{2}{7}, \frac{4}{7}, \frac{6}{7} \}

\[
\begin{align*}
\text{LLL} & \quad \text{Fixed points of } T \\
\text{RRR} & \quad \text{Fixed points of } T^2 \\
\text{RLR} & \quad \text{Fixed points of } T^3 \\
\text{RRR} & \quad \text{Period-4 orbit} \\
\text{RRLR} & \quad \text{Period-4 orbit} \\
\end{align*}
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