Problem 2.

A Bezier cubic curve is defined by the parametric equations
\[
\begin{align*}
    x(t) &= a_xt^3 + b_xt^2 + c_xt + x_1 \\
    y(t) &= a_yt^3 + b_yt^2 + c_yt + y_1
\end{align*}
\]
where \(0 \leq t \leq 1\). The curve goes from \((x(0), y(0)) = (x_1, y_1)\) to \((x(1), y(1)) = (x_4, y_4)\) and is tangent to the lines \((x_1, y_1) - (x_2, y_2)\) and \((x_3, y_3) - (x_4, y_4)\). The Bezier control points are given by the relations
\[
\begin{align*}
    x_2 &= x_1 + c_x/3 \\
    x_3 &= x_2 + (c_x + b_x)/3 \\
    x_4 &= x_1 + c_x + b_x + a_x \\
    y_2 &= y_1 + c_y/3 \\
    y_3 &= y_2 + (c_y + b_y)/3 \\
    y_4 &= y_1 + c_y + b_y + a_y
\end{align*}
\]

(a) Reformulate the problem as a linear system \(Ca = f\) for some matrix \(C\). Write a program to draw a Bezier curve, given control points \((x_1, y_1) \ldots (x_4, y_4)\).

(b) Estimate \(||C||\) via column sums and \(||C^{-1}||\) via choosing \(f\) so that the ratio \(||a||/||f||\) is large, where \(a\) is the solution to \(Ca = f\) above. Compute the condition number using these estimates and compare with the value given by MATLAB condition number estimator \texttt{cond}.

(c) Analyze the problem for accuracy and stability. How many accurate digits can you expect in each calculation? Can you say something about the forward and backward errors in this calculation?

(d) Draw the curve with control points \((0, 0), (2, 1), (-1, 1), (1, 0)\).