The following are some practice problems I would like you to do. Solutions will be discussed in class on 02/01/10.

**Problem 1.**
What do the following pieces of Octave/Matlab code accomplish?
(a) \( x = (0:40)/40; \)
(b) \( a = 2; b = 5; x = a + (b - a) * (0:40)/40; \)
(c) \( x = a + (b - a) * (0:40)/40; y = \sin(x); \text{plot}(x,y); \)

**Problem 2.**
Write the code to implement the factorial function for integers:

\[
\text{function} [\text{nfact}] = \text{factorial}(n)
\]

where \( n \) factorial is equal to \( 1 \cdot 2 \cdot 3 \ldots (n-1) \cdot n \). Either use a ‘for’ loop, or write the function to recursively call itself.

**Problem 3.**
(a) Write a program to compute an approximate value for the derivative of a function using the finite-difference formula

\[
f'(x) \approx \frac{f(x+h) - f(x)}{h}
\]

Test your program using the function \( \tan(x) \) for \( x = 1 \). Determine the error by comparing with the square of the built-in function \( \sec(x) \). Plot the magnitude of the error as a function of \( h \), for \( 10^{-k}, k = 0, \ldots, 16 \). You should use a log scale for \( h \) and for the magnitude of the error. Is there a minimum value for the magnitude of the error? How does the corresponding value of \( h \) compare with the rule of thumb \( h \approx \sqrt{\epsilon_{\text{mach}}} \)? (MATLAB command for \( \epsilon_{\text{mach}} \) is \( \text{eps} \))

(b) Repeat the exercise using the centered difference approximation

\[
f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}
\]

Do you observe any changes compared to the previous case? Explain the behavior of the total error from the point of view of truncation and roundoff errors.

**Problem 4.**
(a) Give an example of floating-point numbers \( x, y, z \) for which addition is not associative, i.e. \( (x + y) + z \neq x + (y + z) \).
(b) Find another example for the multiplication, to show that \( (xy)z \neq x(yz) \).
(c) Finally, find \( x, y, z \) such that multiplication does not distribute over addition: \( x(y + z) \neq xy + xz \).

Avoid using expressions evaluating to NaN or Inf in all of your examples.