# Math 678. Fall 2011.
## Homework #3.
## Due Wednesday 10/12/11 in class.

Solutions should represent individual work, with all necessary details. Only facts discussed in class or given in the main textbook can be used without proof (except the facts known from calculus).

(1) Problem 14, p.87 Evans

(2) Prove that the equation $u_t = u_{xx}, -\infty < x < \infty$ has the following properties:

(a) The translation $u(x-y;t)$ of any solution $u(x;t)$ is another solution for any fixed $y$.

(b) Any derivative ($u_x; u_t; u_{xx}$, etc.) of a solution is again a solution.

(c) A linear combination of solutions is again a solution.

(d) An integral of a solution is again a solution (assuming proper convergence.)

(e) If $u(x;t)$ is a solution, so is the dilated function $u(\sqrt{a}x; at), a > 0$.

(3) Show that there is a domain $V \in \mathbb{R}^n \times (0,1)$ for which Dirichlet problem for the heat equation

$$\begin{cases} u_t = \Delta u, \text{ in } V \\ u = f, \text{ on } \partial V \end{cases}$$

for some continuous $f$ does not have a solution.

(4) Appell's transformation: Let $u(x,t)$ be a solution to the heat equation for $x \in \mathbb{R}, t < 0$. Let $v(x,t) = k(x,t)u(x/t, -1/t)$ for $x \in \mathbb{R}, t > 0$. Show that $v$ solves the heat equation for $x \in \mathbb{R}, t > 0$. This is the analogue of inversion for harmonic functions. *Here* $k(x,t) = (4\pi t)^{-n/2} \exp\left(-|x|^2/4t\right)$ *is the fundamental solution of the heat equation on* $\mathbb{R}^n$.