1. (1.2, #12) Show that the set of even real-valued functions is a vector space under the usual addition and scalar multiplication of functions.

   The standard binary operation of addition and scalar multiplication on the set of all functions satisfies the associative, commutative and distributive properties. In other words, the set of all functions is a vector space over the reals $\mathbb{R}$.. Hence it suffices to show that the set W of even functions is a subspace of the space of all functions. Namely that it is closed under the two operations. Let $f, g \in W$ and $c \in R$, then:

   i) $(f+g)(-x) = f(-x) + g(-x) = f(x) + g(x) = (f+g)(x)$, so $f + g \in W$

   ii) $(cf)(-x) = c(f(-x)) = c(f(x)) = (cf)(x)$, hence $cf \in W$.

   Therefore W is a subspace, in particular it is a vector space.

2. (1.2, #18) Let $V = \{a_1, a_2) : a_1, a_2 \in \mathbb{R}\}$. For $(a_1, a_2), (b_1, b_2) \in V$ and $c \in \mathbb{R}$ define

   $$(a_1, a_2) + (b_1, b_2) = (a_1 + 2b_1, a_2 + 3b_2) \text{ and } c(a_1, a_2) = (ca_1, ca_2)$$

   Is V a vector space over $\mathbb{R}$?

   Proof. It is easy to check that vector addition is not commutative. A specific example (which is what is needed!) where commutivity does not work is given by setting $v_1 = (1, 1)$ and $v_2 = (1, 2)$. Then $v_1 + v_2 = (1, 1) + (1, 2) = (1 + 2, 1 + 6) = (3, 7)$. While $v_2 + v_1 = (1, 2) + (1, 1) = (1+2, 2+3) = (3, 5) \neq v_1 + v_2$. Since all we need is one example where the property fails, V is not a vector space.

3. (1.3, # 12) An $m \times n$ matrix is A called *upper triangular* if all the entries below the diagonal are zero, that is $A_{ij} = 0$ whenever $i > j$. Prove that the set of upper triangular matrices form a subspace of $V = M_{m \times n}(F)$.

   Proof. The most difficult part of this problem is the notation. Let W be the set of all upper triangular matrices in V. For any matric A, we let A_{ij} denote the entry in the ith row and jth column of A Let $A, B \in W$ and let $c \in F$. Thus $A_{ij} = B_{ij} = 0$, whenever $i > j$. Hence $(A+B)_{ij}$ (the ij coordinate of $A+B$) equals $A_{ij} + B_{ij} = 0+0 = 0$ whenever $i > j$. Thus $A + B \in W$. Furthermore $(cA)_{ij}$ (the ij coordinate of cA) equals cA_{ij}. Thus $cA \in W$. Hence W is a subspace.

   Notice that the proof was made easier by giving a name to the subset. Also using the right notation was very helpful!