

**Math 621 HW #4 (Corrected) – Due Wednesday, March 9.**

[8pts]

1. Let  $G$  be a group.

- (a) Show that if every  $a \in G$  satisfies  $a^2 = 1$ , then  $G$  is abelian.
- (b) Show that every group of order 5 or less is abelian.

[6pts]

2. Let  $N$  be the subset of  $S_4$  define as:

$$N := \{(1), (12)(34), (13)(24), (14)(23)\}.$$

Then  $N$  is a subgroup of  $A_4$  - you do not have to prove this! (The only proof of this I see is brute force.) Show that  $N$  is the only non-cyclic subgroup of  $A_4$  of order four. Then show that  $N$  is a normal subgroup of both  $S_4$  and  $A_4$ .

**Additional Problems:**

Sec. 3.4: 5

Sec. 3.5: 3, 14, 15

Sec. 4.3: 2, 5, 10, 25