Work carefully and neatly. You must show all relevant work! You may receive no credit if there is insufficient work.

[3pts] 1. Are the following two graphs homeomorphic? Justify your answer.

\begin{center}
\begin{tikzpicture}
\begin{scope}
\node (A) at (0,0) {A};
\node (B) at (1,0) {B};
\node (C) at (0,-1) {C};
\node (D) at (1,-1) {D};
\node (E) at (0.5,-0.5) {E};
\draw (A) -- (B) -- (E) -- (C) -- (D) -- (A);
\end{scope}
\begin{scope}[shift={(2,0)}]
\node (F) at (0,0) {F};
\node (G) at (1,0) {G};
\node (H) at (0,-1) {H};
\node (I) at (1,-1) {I};
\node (J) at (0.5,-0.5) {J};
\draw (F) -- (G) -- (J) -- (H) -- (I) -- (F);
\end{scope}
\end{tikzpicture}
\end{center}

\textbf{Yes}

Remove vertices \(E+G\) from \(G_2\) (and swing edge over the top)

Remove vertex \(E\) from \(G_1\).

[3pts] 2. Determine if the following graph is planar. Justify your answer.

\begin{center}
\begin{tikzpicture}
\node (A) at (0,0) {A};
\node (B) at (1,0) {B};
\node (C) at (0,-1) {C};
\node (D) at (1,-1) {D};
\node (E) at (0.5,-0.5) {E};
\draw (A) -- (B) -- (C) -- (D) -- (E) -- (A);
\end{tikzpicture}
\end{center}

\textbf{No!}

The vertices \(A, B, D, E, F\) with all edges in \(K_5\)

So by Kuratowski, graph is not planar.

[4pts] 3. Use Euler's formula to show that a tree with \(n\) vertices has \(n-1\) edges.

\textbf{Any tree is planar with 1 region.}

Euler's formula says \(V - E + R = 2\)

If \(V = n, R = 1\). So we have

\[ n - E + 1 = 2 \quad \text{so} \quad n + 1 - 2 = E \]

\[ E = n - 1 \]