Math 108 **Name** (print) KEY

Quiz 7 **Spring 2006**

Work carefully and neatly. You must show all relevant work! You may receive no credit if there is insufficient work. NO GRAPHING CALCULATORS!

[3pts] 1. Find the critical numbers of the function $f(x) = \frac{1}{3}x^3 - 9x + 2$, then classify each critical point as a local max, a local min or neither.

You only have to determine if a critical value gives a max, a min or neither.

Then $f'(x) = x^2 - 9$. Solve $f'(x) = 0$. Then $x^2 - 9 = 0$ factors into $(x+3)(x-3) = 0$. So the solutions are $x = 3$ and $x = -3$.

Plug $-4, 0$ and 4 into $f'(x)$. We get $f'(4) = 16 - 9 = +, f'(0) = -9 = -$ and $f'(4) = 16 - 9 = +$. Thus $x = -3$ gives a local max and $x = 3$ gives a local min.

[4pts] 2. Find the intervals where the function is increasing and where it is decreasing for $f(x) = \dfrac{1}{4 - x^2}$.

Use the quotient rule:

$$f'(x) = \frac{(4 - x^2)0 - (-2x)(1)}{(4 - x^2)^2} = \frac{2x}{(4 - x^2)^2}$$

Thus $f'(x) = 0$ when $x = 0$.
Note that $x = \pm 2$ are not in the domain of the original function $f(x)$. So the real line is broken by the numbers $-2 < 0 < 2$. Plug the numbers $-3, -1, 1, 3$ into $f'(x)$.

Note that the denominator of $f'(x)$ is squared, so it is always positive. Thus the sign of $f'(x)$ is equal to the sign of the numerator of $f'(x)$. Then $f'(-3) = -, f'(-1) = -, f'(1) = +, f'(3) = +$. Hence the function is decreasing on $(-\infty, 0)$ (or $x < 0$) and increasing on $(0, \infty)$ (or $x > 0$).

[3pts] 3. Use the second derivative test to find the relative maxima and minima of the function $f(x) = x^4 - 2x^2 + 3$.

The critical numbers are the solutions to $f'(x) = 0$. Then $f'(x) = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x+1)(x-1)$. The roots are $x = 0, x = -1, x = 1$. Plug these number into $f''(x) = 12x^2 - 4$. Then $f''(0) = -4 = -, f''(-1) = 12 - 4 = +, f''(1) = 12 - 4 = +$. Thus by the second derivative test, $x = 0$ gives a local max, $x = \pm 1$ give local mins.