Fall 2023 - Math 685 Section 001
Numerical Analysis

Lectures: Mon - Wed, 3:00-4:15PM, Expl Hall 4106

Prerequisites. Undergraduate linear algebra, differential equations and some familiarity with MATLAB.

Course description: This course is an introduction to four basic topics in numerical analysis and two additional topics which deals with numerical methods for initial value problems for ordinary differential equations and boundary value problems (mostly in 1D).

1. Interpolation and Approximation Theory (4 weeks)
   (a) Lagrange interpolation, Newton form, divided differences
   (b) Error analysis, Chebyshev polynomials, Jackson Theorm
   (c) Uniform Approximation, Weierstrass's Theorem and Bernstein polynomials
   (d) Piecewise polynomial interpolation (Lagrange and Hermite), cubic splines, error analysis
   (e) Least squares, normal equations, orthogonal polynomials, QR factorization, singular value decomposition.
   (f) Approximation properties of deep neural networks [1]
2. Quadrature (2 weeks)
   (a) Peano Kernel and Euler-Maclaurin expansion
   (b) Extrapolation and Romberg integration
   (c) Adaptive quadrature
   (d) Gaussian quadrature.
3. Iterative Methods for Linear Systems of Equations (3 weeks)
   (a) Classical methods: Jacobi, Gauss-Seidel, SOR, convergence
   (b) Krylov sequence methods, conjugate gradient method, GMRES, preconditioning, convergence.
4. Eigenvalue Problems (3 weeks)

[1] If time permits

(a) Similarity transformations
(b) Rayleigh quotients
(c) Power and inverse power methods
(d) Householder transformations
(e) QR algorithm, shift and deflation, convergence
(f) Singular value decomposition

5. Numerical Methods for Initial Value Problems (2 weeks)
   (a) Runge-Kutta methods
   (b) Multistep methods
   (c) Consistency, stability and convergence analysis
   (d) Error estimation and stepsize control
   (e) Methods for stiff systems.

6. Numerical Solutions for Boundary Value Problems (1 week)
   (a) Two-point boundary value problems
   (b) Finite difference methods
   (c) Variational formulation and the finite element method
   (d) Introduction to error analysis
   (e) Discretization methods for multidimensional problems.

Text: There is no need to buy any textbook but here are few references which will be useful [8, 5, 1, 9, 3, 2, 7, 6, 4].

Exams (50%): There will be a MIDTERM exam and will amount to 20% of the grade. The FINAL exam will be a comprehensive and will constitute 30% of the grade.

Homework (50%): There will be about 6 HOMEWORKS which will amount to 50% of the final grade. The homeworks will be about 80% theoretical and 20% computational using MATLAB. There will be a penalty of 10% per day late; homeworks will not be accepted after one week.

Students are encouraged to work in groups but must hand in an individual self written proofs and answers.

Midterm Policy: Make-up examinations will be given *only* in the case of an absence caused by illness, participation in a University activity at the request of of the University authorities, or compelling circumstances beyond the students' control. Convincing documentation such as a doctor's note will be required.

The reason for absence must to unavoidable, documented, and reported to me at least 48 hours before the exam date. The instructor will determine if the excuse is valid.

Academic Integrity: GMU is an Honor Code university; please see the Office for Academic Integrity for a full description of the code and the honor committee process. The principle of academic integrity is taken very seriously and violations are treated gravely. What does academic integrity mean in this course? Essentially this: when you are responsible for a task, you will perform that task. When you rely on someone else's work in an aspect of the performance of that task, you will give full credit in the proper, accepted form. Another aspect of academic integrity is the free play of ideas. Vigorous discussion and debate are encouraged in this course, with the firm expectation that all aspects of the class will be conducted with civility and respect for differing ideas, perspectives, and traditions. When in doubt (of any kind) please ask for guidance and clarification.

Mason email accounts. Students must use their Mason email account for any correspondence during this course. For more information see: `http://masonlive.gmu.edu`.

Office of Disability Services. If you are a student with a disability and you need academic accommodations, please see me and contact the Office of Disability Services (ODS) at 993-2474, `http://ods.gmu.edu`. All academic accommodations must be arranged through the ODS.

University policies The University Catalog, `http://catalog.gmu.edu`, is the central resource for university policies affecting student, faculty, and staff conduct in university academic affairs. Other policies are available at `http://universitypolicy.gmu.edu`. All members of the university community are responsible for knowing and following established policies.

References

[1] K. E. Atkinson. *An introduction to numerical analysis.* John Wiley & Sons Inc., New York, second edition, 1989.

[2] P.J. Davis. *Interpolation and approximation.* Dover Publications, 1975.

[3] G. H. Golub and C. F. Van Loan. *Matrix computations.* Johns Hopkins Studies in the Mathematical Sciences. Johns Hopkins University Press, Baltimore, MD, third edition, 1996.

[4] Philipp Christian Petersen. Neural network theory. *University of Vienna*, 535, 2020.

[5] A. Quarteroni, R. Sacco, and F. Saleri. *Numerical mathematics*, volume 37 of *Texts in Applied Mathematics.* Springer-Verlag, Berlin, second edition, 2007.

[6] L. F. Shampine. *Numerical solution of ordinary differential equations.* Chapman & Hall, New York, 1994.

[7] G. W. Stewart. *Afternotes goes to graduate school.* Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1998. Lectures on advanced numerical analysis.

[8] J. Stoer and R. Bulirsch. *Introduction to numerical analysis*, volume 12 of *Texts in Applied Mathematics.* Springer-Verlag, New York, third edition, 2002. Translated from the German by R. Bartels, W. Gautschi and C. Witzgall.

[9] E. Süli and D. F. Mayers. *An introduction to numerical analysis.* Cambridge University Press, Cambridge, 2003.