Now Try Exercise 33

PRACTICE PROBLEMS 12.4

1. Prove $(p \to q) \Leftrightarrow (\sim p \vee q)$.
2. Rewrite the statement
$$(p \to \sim q) \to (r \wedge p)$$
eliminating the connective $\to$ and simplifying if possible.
3. Assume that the statement "Jim goes to the ballgame only if Ted gets tickets" is TRUE.
   (a) Write the contrapositive and give its truth value.
   (b) Write the converse and give its truth value.
   (c) Write the negation and give its truth value.

EXERCISES 12.4

1. Show that $[(p \to q) \wedge q] \to p$ is not a tautology.
2. Show that the distributive laws hold:
   (a) $[p \vee (q \wedge r)] \Leftrightarrow [(p \vee q) \wedge (p \vee r)]$
   (b) $[p \wedge (q \vee r)] \Leftrightarrow [(p \wedge q) \vee (p \wedge r)]$.
3. Show that the second law of implication holds:
$$(p \to q) \Leftrightarrow \sim(p \wedge \sim q).$$
4. Write a statement equivalent to
$$\sim(p \wedge \sim q) \to (r \wedge p)$$
using only $\sim$ and $\wedge$.
5. The Sheffer stroke is defined by the truth table

p	q	$p\|q$
T	T	F
T	F	T
F	T	T
F	F	T

   (a) Show that $\sim p \Leftrightarrow p|p$.
   (b) Show that $p \vee q \Leftrightarrow (p|p)|(q|q)$.
   (c) Show that $p \wedge q \Leftrightarrow (p|q)|(p|q)$.
   (d) Write $p \to q$ using only the Sheffer stroke.
   (e) Write $p|q$ using only $\sim$ and $\wedge$.
6. Without using truth tables, show that
$$[(p \vee \sim q) \wedge r] \to p \Leftrightarrow (p \vee q) \vee \sim r.$$
7. Show that
$$(p \to q) \Leftrightarrow [(p \wedge \sim q) \to c]$$
using the fact that c denotes a contradiction; that is, c is always FALSE. Read the statement aloud.
8. (a) Prove that
$$p \Rightarrow [q \to (p \wedge q)].$$
   (b) True or false?
$$p \Leftrightarrow [q \to (p \wedge q)]$$
9. True or false?
$$(p \vee q) \Rightarrow [q \to (p \wedge q)]$$
10. True or false?
$$[p \vee (p \wedge q)] \Leftrightarrow p$$
11. Write an equivalent form of $p \oplus q$ using only $\sim$ and $\vee$.
12. Write each statement using only the connectives $\sim$ and $\vee$.
   (a) $p \wedge \sim q \to p$ (b) $(p \to r) \wedge (q \vee r)$
   (c) $p \to [r \wedge (p \vee q)]$ (d) $(p \wedge q) \to (\sim q \vee r)$
13. Write an equivalent form of the statement below without using $\to$.
$$(p \vee q) \to (q \wedge \sim r)$$
14. Write an equivalent form of the statement below without using $\to$.
$$(q \to \sim r) \to (p \wedge q)$$
15. Write an equivalent form of the statement below without using $\to$.
$$\sim(p \wedge \sim q) \to (p \vee \sim r)$$
16. Write an equivalent form of the statement below without using $\to$.
$$\sim(\sim(p \to q) \to r)$$
17. Negate the following statements.
   (a) Arizona borders California and Arizona borders Nevada.
   (b) There are tickets available or the agency can get tickets.
   (c) The killer's hat was either white or gray.

18. Negate the following statements.
    (a) Montreal is a province in Canada and Ottawa is a province in Canada.
    (b) The salesman goes to the customer or the customer calls the salesman.
    (c) The hospital does not admit psychiatric patients or orthopedic patients.
19. Use DeMorgan's laws to negate the statement $p \vee \sim q \wedge r$ without using parentheses.
20. Use DeMorgan's laws to negate the statement $(p \vee \sim q) \vee (\sim r)$ without using parentheses.
21. Negate each of the statements.
    (a) Either Jeremy takes 12 credits or Jeremy takes 15 credits this semester.
    (b) Sandra received a gift from Sally and a gift from Sacha.
22. Negate the following statements.
    (a) The plane from California to Maine was on time and every seat on it was taken.
    (b) Kenneth was not on time nor was he dressed properly.
23. Write each statement as an OR or as an AND statement and then write its negation.
    (a) "The Old Man and the Sea" was written by Ernest Hemingway or by Jack London.
    (b) "H. M. S. Pinafore" was written by Gilbert and Sullivan.
24. Negate each of the following statements.
    (a) Issac Newton and Henry Ford invented the calculus.
    (b) James Galway or Paul Simon plays the piano.
25. Negate the following statements.
    (a) If I have a ticket to the theater, I spent a lot of money.
    (b) Basketball is played on an indoor court only if the players wear sneakers.
    (c) The stock market is going up implies that the interest rates are going down.
    (d) For humans to stay healthy, it is sufficient that humans have enough water.
26. For each of the following statements, give the contrapositive and determine its truth value.
    (a) If the lists in the book are not in alphabetical order, it is not a telephone directory.
    (b) The water is salty if it comes from the Atlantic Ocean.
27. For each of the following statements, give the contrapositive and determine its truth value.
    (a) If the sum of odd numbers is odd, then the number of items in the sum is odd.
    (b) If the computer keyboard is standard, then "K" is not next to "W."
28. For each statement, give the contrapositive, the converse, and the negation. Determine the truth value in each case.
    (a) If a rectangle has equal sides, it is a square.
    (b) An airplane flies faster than the speed of sound only if it is a Concorde.
    (c) If the intersection of two sets is not empty, then the union of the two sets is not empty.
    (d) If a coin is fair, the probability of a head is $\frac{1}{2}$.
29. Give the contrapositive and the converse of each statement and then give the truth value of each.
    (a) If a bird is small, it is a hummingbird.
    (b) If two nonvertical lines have the same slope, they are parallel.
    (c) If we are in Paris, we must be in France.
    (d) If a road is one-way, you cannot legally make a U-turn.
30. Bill, Sue, and Alice are lined up facing forward with Bill first, then Sue, then Alice. From a collection that they know contains three blue and two red hats, hats are placed on their heads while they are blindfolded. Blindfolds are removed, but they continue to face forward and see only those hats in front of them. Bill sees none, Sue sees Bill's, and Alice sees both Sue's and Bill's. Alice claims she does not know what color her hat is. Sue also claims that she does not know what color her hat is. Now Bill knows what color his hat is. What is it?
31. A prisoner is given one chance for freedom. He may ask one yes-or-no question of either of two guards. Each guard allows access to one of two unmarked doors. One is the door to freedom, the other the door to death. One guard always tells the truth, the other always lies. What question should the prisoner ask?
32. Verify modus tollens (logical implication 4 in Table 2).
33. Verify hypothetical syllogism (logical implication 6 in Table 2).