Using the table below, find the inverse Laplace transform of the function

\[ F(s) = \frac{3s}{s^2 - s - 6}. \]

Solution: We need to use partial fractions to rewrite \( F(s) \) in a form such that we can use the table.

\[
\frac{3s}{s^2 - s - 6} = \frac{3s}{(s - 3)(s + 2)} = \frac{A}{s - 3} + \frac{B}{s + 2} = \frac{A(s + 2) + B(s - 3)}{(s - 3)(s + 2)}.
\]

Therefore we must have

\[ A(s + 2) + B(s - 3) = 3s. \]

Plugging in \( s = 3 \) gives \( 5A = 9 \) or \( A = 9/5 \), and plugging in \( s = -2 \) gives \( -5B = -6 \) or \( B = 6/5 \). Therefore we have that

\[ F(s) = \frac{3s}{s^2 - s - 6} = \frac{9}{5} \frac{1}{s - 3} + \frac{6}{5} \frac{1}{s + 2}, \]

so that

\[
\mathcal{L}^{-1}(F(s)) = \frac{9}{5} \mathcal{L}^{-1} \left( \frac{1}{s - 3} \right) + \frac{6}{5} \mathcal{L}^{-1} \left( \frac{1}{s + 2} \right).
\]

Consulting the table gives the answer

\[ \mathcal{L}^{-1}(F(s)) = \frac{9}{5} e^{3t} + \frac{6}{5} e^{-2t}. \]

Using the table below, find the inverse Laplace transform of the function

\[ F(s) = \frac{2s - 3}{s^2 - 4}. \]

Solution: We need to use partial fractions to rewrite \( F(s) \) in a form such that we can use the table.

\[
\frac{2s - 3}{s^2 - 4} = \frac{2s - 3}{(s - 2)(s + 2)} = \frac{A}{s - 2} + \frac{B}{s + 2} = \frac{A(s + 2) + B(s - 2)}{(s - 2)(s + 2)}.
\]
Therefore we must have
\[ A(s + 2) + B(s - 2) = 2s - 3. \]
Plugging in \( s = 2 \) gives \( 4A = 1 \) or \( A = 1/4 \), and plugging in \( s = -2 \) gives \(-4B = -7\) or \( B = 7/4 \). Therefore we have that
\[
F(s) = \frac{2s - 3}{s^2 - 4} = \frac{1}{4} s - 2 + \frac{7}{4} s + 2
\]
so that
\[
\mathcal{L}^{-1}(F(s)) = \frac{1}{4} \mathcal{L}^{-1} \left( \frac{1}{s - 2} \right) + \frac{7}{4} \mathcal{L}^{-1} \left( \frac{1}{s + 2} \right).
\]
consulting the table gives the answer
\[
\mathcal{L}^{-1}(F(s)) = \frac{1}{4} e^{2t} + \frac{7}{4} e^{-2t}.
\]

Using the table below, find the inverse Laplace transform of the function
\[ F(s) = \frac{s + 1}{(s - 2)^2}. \]

Solution: We need to use partial fractions to rewrite \( F(s) \) in a form such that we can use the table.
\[
\frac{s + 1}{(s - 2)^2} = \frac{A}{s - 2} + \frac{B}{(s - 2)^2} = \frac{A(s - 2) + B}{(s - 2)^2}.
\]
Therefore we must have
\[ A(s - 2) + B = s + 1. \]
Plugging in \( s = 2 \) gives \( B = 3 \), and plugging in, say, \( s = 0 \) gives \(-2A + 3 = 1 \) or \( A = 1 \) (I could also have taken the first derivative giving directly \( A = 1 \)). Therefore we have that
\[
F(s) = \frac{s + 1}{(s - 2)^2} = \frac{1}{s - 2} + \frac{3}{(s - 2)^2}
\]
so that
\[
\mathcal{L}^{-1}(F(s)) = \mathcal{L}^{-1} \left( \frac{1}{s - 2} \right) + 3 \mathcal{L}^{-1} \left( \frac{1}{(s - 2)^2} \right).
\]
consulting the table gives the answer
\[
\mathcal{L}^{-1}(F(s)) = e^{2t} + 3 t e^{2t}.
\]