Use the method of undetermined coefficients to find a particular solution to the differential equation.

\[ y'' + 2y' + y = 2e^{-t}. \]

(Hint: A fundamental system for the homogeneous equation is \( \{ e^{-t}, te^{-t} \} \).)

Solution: Since the right hand side of the equation is a solution to the homogeneous equation, and since \( t \) multiplied by it is also a solution, we look for a particular solution of the form \( Y(t) = At^2 e^{-t} \). We have

\[
Y'(t) = -A t^2 e^{-t} + 2A t e^{-t}
\]
\[
Y''(t) = At^2 e^{-t} - 4At e^{-t} + 2A e^{-t}.
\]

Plugging into the original equation gives

\[
At^2 e^{-t} - 4At e^{-t} + 2A e^{-t} - 2At^2 e^{-t} + 4At e^{-t} + At^2 e^{-t} = 2e^{-t}.
\]

Canceling terms gives

\[
2A e^{-t} = 2e^{-t}
\]

so that \( A = 1 \). Therefore, a particular solution is

\[ Y(t) = t^2 e^{-t}. \]

Use the method of undetermined coefficients to find a particular solution to the differential equation.

\[ y'' + y' + y = t^2 + t. \]

(Hint: It will be enough to look for a particular solution in the form of a general quadratic polynomial.)

Solution: Following the hint, we look for a particular solution of the form \( Y(t) = At^2 + Bt + C \). We have

\[
Y'(t) = 2At + B
\]
\[
Y''(t) = 2A.
\]
Plugging into the original equation gives
\[ 2A + 2At + B + At^2 + Bt + C = t^2 + t. \]
Gathering like terms gives
\[ At^2 + (2A + B)t + (2A + B + C) = t^2 + t \]
which leads to
\[
\begin{align*}
A &= 1 \\
2A + B &= 1 \\
2A + B + C &= 0
\end{align*}
\]
The first equation gives \( A = 1 \), the second, \( B = -1 \), and the third, \( C = -1 \). Therefore, a particular solution is
\[ Y(t) = t^2 - t - 1. \]

Use the method of undetermined coefficients to find a particular solution to the differential equation.

\[ y'' + 2y' = 4 \sin(2t). \]

Solution: We look for a particular solution of the form \( Y(t) = A \cos(2t) + B \sin(2t) \). We have
\[
\begin{align*}
Y'(t) &= -2A \sin(2t) + 2B \cos(2t) \\
Y''(t) &= -4A \cos(2t) - 4B \sin(2t).
\end{align*}
\]
Plugging into the original equation gives
\[ -4A \cos(2t) - 4B \sin(2t) - 4A \sin(2t) + 4B \cos(2t) = 4 \sin(2t). \]
Gathering like terms gives
\[ (-4A + 4B) \cos(2t) + (-4A - 4B) \sin(2t) = 4 \sin(2t) \]
which leads to
\[
\begin{align*}
-4A + 4B &= 0 \\
-4A - 4B &= 4
\end{align*}
\]
The first equation gives \( A = B \), and the second, \( -8B = 4 \), so that \( B = -1/2 \) and \( A = -1/2 \). Therefore, a particular solution is
\[ Y(t) = \frac{1}{2} \cos(2t) - \frac{1}{2} \sin(2t). \]