Solve the following exact equation: \((2xy^2 + 2y) + (2x^2y + 2x)y' = 0\).

Solution: We are looking for a function \(\Psi(x, y)\) such that \(\Psi_x = 2xy^2 + 2y\) and \(\Psi_y = 2x^2y + 2x\). This leads to

\[
\begin{align*}
\Psi_x &= 2xy^2 + 2y \\
\Psi &= x^2y^2 + 2xy + g(y) \\
\Psi_y &= 2x^2y + 2x + g'(y).
\end{align*}
\]

This leads to

\[
\begin{align*}
2x^2y + 2x + g'(y) &= 2x^2y + 2x \\
g'(y) &= 0 \\
g(y) &= 0.
\end{align*}
\]

Therefore \(\Psi = x^2y^2 + 2xy\) and the solution to the ODE is given by

\[x^2y^2 + 2xy = c.\]

Solve the following exact equation: \((3x^2 - 2xy) + (6y^2 - x^2)y' = 0\).

Solution: We are looking for a function \(\Psi(x, y)\) such that \(\Psi_x = 3x^2 - 2xy\) and \(\Psi_y = 6y^2 - x^2\). This leads to

\[
\begin{align*}
\Psi_x &= 3x^2 - 2xy \\
\Psi &= x^3 - x^2y + g(y) \\
\Psi_y &= -x^2 + g'(y).
\end{align*}
\]

This leads to

\[
\begin{align*}
-x^2 + g'(y) &= 6y^2 - x^2 \\
g'(y) &= 6y^2 \\
g(y) &= 2y^3.
\end{align*}
\]

Therefore \(\Psi = x^3 - x^2y + 2y^3\) and the solution to the ODE is given by

\[x^3 - x^2y + 2y^3 = c.\]
Solve the following exact equation: 

\[(9x^2 + y - 1) + (x - 4y)y' = 0.\]

Solution: We are looking for a function \(\Psi(x, y)\) such that \(\Psi_x = 9x^2 + y - 1\) and \(\Psi_y = x - 4y\). This leads to

\[
\begin{align*}
\Psi_x &= 9x^2 + y - 1 \\
\Psi &= 3x^3 + xy - x + g(y) \\
\Psi_y &= x + g'(y).
\end{align*}
\]

This leads to

\[
\begin{align*}
x + g'(y) &= x - 4y \\
g'(y) &= -4y \\
g(y) &= -2y^2.
\end{align*}
\]

Therefore \(\Psi = 3x^3 + xy - x - 2y^2\) and the solution to the ODE is given by

\[3x^3 + xy - x - 2y^2 = c.\]