Solve the following initial value problem: \( y' + 3y = 1 + e^{-2t}, \ y(0) = 1. \)

Solution: The integrating factor in this case is \( \mu(t) = e^{\int 3dt} = e^{3t} \) which leads to the following solution.

\[
e^{3t} y' + 3e^{3t}y = e^{3t}(1 + e^{-2t})
\]

\[
\frac{d}{dt}(e^{3t} y) = e^{3t} + e^t
\]

\[
e^{3t}y = \frac{1}{3}e^{3t} + e^t + c
\]

\[
y = \frac{1}{3} + e^{-2t} + ce^{-3t}
\]

\( y(0) = 1 \) leads to \( 1 = 1/3 + 1 + c \) or \( c = 1/3. \) Hence the final solution is

\[
y = \frac{1}{3} + e^{-2t} + \frac{1}{3}e^{-3t}.
\]

Solve the following initial value problem: \( y' - 2y = t^2 e^{2t}, \ y(0) = 1. \)

Solution: The integrating factor in this case is \( \mu(t) = e^{\int -2dt} = e^{-2t} \) which leads to the following solution.

\[
e^{-2t} y' - 2e^{-2t}y = e^{-2t}(t^2 e^{2t})
\]

\[
\frac{d}{dt}(e^{-2t} y) = t^2
\]

\[
e^{-2t}y = \frac{1}{3}t^3 + c
\]

\[
y = \frac{1}{3}e^{2t} t^3 + ce^{-2t}
\]

\( y(0) = 1 \) leads to \( 1 = 0 + c \) or \( c = 1. \) Hence the final solution is

\[
y = \frac{1}{3}e^{2t} t^3 + e^{2t}.
\]

Solve the following initial value problem: \( y' + y = te^{-t} + 1, \ y(0) = 2. \)
Solution: The integrating factor in this case is $\mu(t) = e^{\int dt} = e^t$ which leads to the following solution.

\[
\begin{align*}
\quad e^t y' + e^t y &= e^t(te^{-t} + 1) \\
\frac{d}{dt}(e^t y) &= t + e^t \\
\quad e^t y &= \frac{1}{2} t^2 + e^t + c \\
\quad y &= \frac{1}{2} e^{-t} t^2 + 1 + c e^{-t}
\end{align*}
\]

$y(0) = 2$ leads to $2 = 0 + 1 + c$ or $c = 1$. Hence the final solution is

\[
y = \frac{1}{2} e^{-t} t^2 + 1 + e^{-t}.
\]