Exam 1 - Wed 9/23 - Cover Ch 12
1. One 3 x 5 card with notes (both sides)
2. No calculators permitted.

(2.5.57) \( x + y + z = 1 \quad x + y = 2 \)

DIRECTION VECTOR

\[ \vec{v} = \vec{u}_1 \times \vec{u}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -\vec{j} + \vec{k} \]

\[ \vec{u}_1 = \langle 1, 1, 1 \rangle \quad \vec{u}_2 = \langle 1, 1, 0 \rangle \]

Set \( x = 0 \): \[ y + z = 1 \quad z = -1 \]
\[ y = 2 \]

Point: \( (0, 2, -1) \)

Parametric Equations:
\[ x = t \]
\[ y = 2 + t \]
\[ z = -1 \]
\[45\] \[x + 2y + 6z = 1\]
\[x + 2y + 6z = 10\]

1. Find point on \(x + 2y + 6z = 1\)
2. Find dist of point to plane \(x + 2y + 6z = 10\)
3. Find param eqn of line through \(P\) with dir vector \(\vec{v}\).
4. Find when that line intersects plane.

\[\vec{P_0P} = \langle -9, \theta, 0 \rangle\]
\[\text{dist: } \frac{|\vec{P_0P} \cdot \vec{n}|}{|\vec{n}|} = \frac{|-9|}{\sqrt{1+4+36}} = \frac{9}{\sqrt{41}}\]

\[53\] \[x = 1 - t\]
\[y = 3t\]
\[z = 1 + t\]
\[2x - y + 3z = 6 \Rightarrow 2(1-t) - 3t + 3(1+t) = 6\]
\[2 - 2t - 3t + 3 + 3t = 6\]
\[-2t = 1\]
\[t = -\frac{1}{2}\]

\(x = \frac{9}{2}\)
\(y = -\frac{3}{2}\)
\(z = \frac{1}{2}\)
Find some $P_0$ on $L$
Set $t=0$: $(0,0,0)=P_0$

$\vec{P_0P} = \langle 0,0,12 \rangle$

$\vec{u} = \langle 4, -2, 2 \rangle$

\[
\text{dist} = \frac{|\vec{P_0P} \times \vec{u}|}{|\vec{u}|}
\]

\[
\vec{P_0P} \times \vec{u} = \left( \frac{12}{2} \right) \times \left( 4 \hat{i} - 2 \hat{j} + 2 \hat{k} \right)
\]

\[
= 48 \left( \hat{i} \times \hat{i} \right) - 24 \left( \hat{j} \times \hat{j} \right) + 24 \left( \hat{k} \times \hat{k} \right)
\]

\[
= 48 \hat{j} + 24 \hat{k}
\]
3.1 Vector Functions

\[ y = f(x) \quad f: \mathbb{R} \to \mathbb{R} \]
\[ y \in \mathbb{R} \times \mathbb{R} \]

Look at \( f: \mathbb{R} \to \mathbb{R}^3 \)

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real 3-vectors

We write \( \mathbf{r}(t) \) for a vector function that maps real numbers to vectors (2 or 3-dim)

Write \( \mathbf{r}(t) = f(t) \mathbf{i} + g(t) \mathbf{j} + h(t) \mathbf{k} \)

\( f, g, h \) are ordinary functions called component functions

Also write \( \mathbf{r}(t) = x(t) \mathbf{i} + y(t) \mathbf{j} + z(t) \mathbf{k} \)

The graph of \( \mathbf{r}(t) \) is the parametric curve

\[ x = f(t) \]
\[ y = g(t) \]
\[ z = h(t) \]
19 Circular motion

\[ \mathbf{v}(t) = \cos t \mathbf{i} + \sin t \mathbf{j} \]

\[ \mathbf{v}(0) = \mathbf{i} \]
\[ \mathbf{v}(\pi) = \frac{\sqrt{2}}{2} \mathbf{i} + \frac{\sqrt{2}}{2} \mathbf{j} \]
\[ \mathbf{v}(\pi) = -\mathbf{i} \]

Note: 1. \( \cos^2 t + \sin^2 t = 1 \) so \( |\mathbf{v}(t)| = 1 \) all \( t \)
2. \( \mathbf{v}(t) \) has a direction and sense of movement so we can talk about velocity, acceleration, and speed
3. The curve traced out by \( \mathbf{v}(t) \) is independent of the motion