Answer each of the following questions. Show all work, as partial credit may be given. This exam is out of a total of 80 points.

1. (8 pts.) Find \( f_x, f_y, f_{yy}, \) and \( f_{xy} \) for the function \( f(x, y) = x \sin(xy) \).

2. (8 pts.) Find \( f_{yzy} \) when \( f(x, y, z) = y^2 z^2 + x^3 y + \frac{xy}{z} \).

3. (8 pts. each) Let \( f(x, y) = x^2 y^2 - 2x^3 y + 2x \).
   (a) Find \( \nabla f \).
   (b) Find the directional derivative of \( f \) at the point \((1, 2)\) and in the direction \( \mathbf{v} = i + 3j \).
   (c) Find the maximum rate of change of \( f \) at the point \((1, 2)\), and the direction in which \( f \) changes most rapidly at the point \((1, 2)\). (Note: Direction should be in the form of a unit vector.)
   (d) Find the linearization of the function \( f(x, y) \) at the point \((1, 2)\).
   (e) Use differentials to estimate the change in \( f \) when the point \((1, 2)\) moves to the point \((1.1, 2.3)\).

4. (12 pts.) Find all critical points of the function \( f(x, y) = 4xy - x^4 - y^4 \) and identify each as a local maximum, local minimum, or saddle point. (Hint: There are three critical points.)

5. (6 pts.) Evaluate the iterated integral \( \int_0^1 \int_x^3 (x + y) \, dy \, dx \).

6. (6 pts.) Reverse the order of integration for the integral \( \int_0^2 \int_0^{x^2} x^2 \, dy \, dx \). Do not evaluate. (Hint: It will be helpful to sketch the region before reversing the order.)