MATH 213 – 14 OCTOBER 2009 – EXAM 2

Answer each of the following questions. Show all work, as partial credit may be given. This exam is counted out of a total of 80 points.

1. Consider the vector function \( \mathbf{r}(t) = 3t^2 \mathbf{i} + t^3 \mathbf{j} + 6t \mathbf{k} \).
   
   (a) (12 pts.) If \( \mathbf{r}(t) \) is interpreted as the position of a particle at time \( t \), find the velocity, acceleration, and speed of the particle.
   
   (b) (6 pts.) Find the unit tangent vector of \( \mathbf{r}(t) \).

2. Consider the curve given by \( \mathbf{r}(t) = 5t^2 \mathbf{i} + 4t^2 \mathbf{j} + (3 - 3t^2) \mathbf{k}, \quad 0 \leq t \leq 1 \).
   
   (a) (12 pts.) Find the arclength of the given curve.
   
   (b) (6 pts.) Compute the arclength parameter, \( s(t) \), for the given curve.

3. (8 pts. each) A projectile is fired horizontally with a velocity of 180 feet per second from an altitude of 100 feet above level ground.
   
   (a) Find a vector function giving the position of the projectile \( t \) seconds after it has been fired.
   
   (b) When and how far downrange does the projectile strike the ground? Be sure to express your answers in correct units.
   
   (c) What is the speed of the projectile when it strikes the ground? Be sure to express your answer in correct units.

4. (10 pts.) Sketch the level curves for the function \( f(x, y) = x^2 - y \) corresponding to the values \( z = 0, \ z = 1, \) and \( z = 2 \). Label at least one point on each of the level curves.

5. (10 pts.) Show that the limit \( \lim_{(x,y) \to (0,0)} \frac{2x^2 - y^2}{x^2 + 2y^2} \) does not exist by finding two paths along which the limits differ.