Answer each of the following questions. Show all work, as partial credit may be given. This exam is out of a total of 50 points.

1. (10 pts.) Let $H$ be the subset of $P_3$ (the set of all polynomials of degree at most 3) of polynomials of the form $p(t) = a + (a + b)t + bt^2$, where $a, b$ are in $\mathbb{R}$. Show that $H$ is a subspace of $P_3$ by finding a spanning set for $H$.

2. (10 pts.) Let $B = \left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -5 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -7 \\ 3 \end{bmatrix} \right\}$. Show that $B$ is a basis for $\mathbb{R}^3$. In other words, verify that $B$ satisfies both parts of the definition of a basis.

3. Let $A = \begin{bmatrix} 1 & 0 & -5 & 1 & 4 \\ -2 & 1 & 6 & -1 & -2 \\ 0 & 2 & -8 & 2 & 12 \end{bmatrix}$.

   (a) (5 pts.) Find $k$ such that $Nul(A)$ is a subspace of $\mathbb{R}^k$ and find $k$ such that $Col(A)$ is a subspace of $\mathbb{R}^k$.

   (b) (15 pts.) Find bases for $Nul(A)$ and $Col(A)$.

4. (5 pts. each) Let $B = \{b_1, b_2\}$ be a basis for $\mathbb{R}^2$ where $b_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and $b_2 = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$.

   (a) Find $x$ if $[x]_B = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$.

   (b) Find $[x]_B$ if $x = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. 