Answer each of the following questions. Show all work, as partial credit may be given. This exam is out of a total of 70 points.

1. Consider the linear system
   \[ \begin{align*}
   x_1 + 3x_2 - 5x_3 &= 4 \\
   x_1 + 4x_2 - 8x_3 &= 7 \\
   -3x_1 - 7x_2 + 9x_3 &= -6
   \end{align*} \]
   (a) (5 pts.) Write down the augmented matrix for this system.
   (b) (5 pts.) Write down the vector equation equivalent to this system.
   (c) (5 pts.) Write down the matrix equation equivalent to this system.
   (d) (10 pts.) Solve the system and describe all solutions in parametric vector form.

2. (5 pts. each) Let \( A = \begin{bmatrix} -5 & 7 & 9 \\ 1 & -2 & 6 \end{bmatrix} \).
   (a) Using elementary row operations, find the echelon form and the reduced echelon form for \( A \).
   (b) Find all solutions to the homogeneous equation \( Ax = 0 \) in parametric vector form.

3. Let \( A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -2 & -4 \end{bmatrix} \).
   (a) (5 pts.) With the least amount of work possible, show that the columns of \( A \) span a plane in \( \mathbb{R}^3 \).
   (b) (10 pts.) Is the vector \( \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} \) in the span of the columns of \( A \)? Fully justify your answer.

4. (10 pts.) Do the columns of the matrix \( \begin{bmatrix} 0 & 4 & 5 \\ 1 & 6 & -1 \\ -3 & 6 & -9 \end{bmatrix} \) span all of \( \mathbb{R}^3 \)? Fully justify your answer.

5. (10 pts. each) Let \( A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & -1 & 4 \\ 5 & 4 & -1 \\ -4 & -3 & 0 \end{bmatrix} \)
   (a) Show that the columns of \( A \) form a linearly dependent set.
   (b) Find a dependence relation for the columns of \( A \).