Answer each of the following questions. Show all work, as partial credit may be given.

1. (6 pts. each) Let \( W = \left\{ \begin{bmatrix} 2s + 3t \\ r + s - 2t \\ 4r + s \\ 3r - s - t \end{bmatrix} : r, s, t \text{ real} \right\} \).

(a) Find a set \( S \) of vectors that spans \( W \).

(b) Is the set you found in part (a) a basis for \( W \)? Fully justify your answer.

2. Let \( B = \{v_1, v_2, v_3\} \) where
\[
 v_1 = \begin{bmatrix} 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad \text{and} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \end{bmatrix}
\]
be a a basis for \( H = \text{Span}\{v_1, v_2, v_3\} \).

(a) (4 pts.) Find a number \( a \) so that \( H \) is a subspace of \( \mathbb{R}^a \), and find \( \dim H \).

(b) (6 pts.) If \( [x]_B = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \), find \( x \).

(c) (6 pts.) If \( x = \begin{bmatrix} -1 \\ 6 \\ 0 \\ 0 \end{bmatrix} \), find \( [x]_B \).

3. (6 pts.) Determine if the set \( \{t^2 + 1, 1 - t^2, 1 + t\} \) is a basis for \( \mathbb{P}_2 \), the vector space of polynomials of degree 2 or less. Fully justify your answer. Use any method you like.

4. Consider the matrix
\[
 \begin{bmatrix} 1 & -3 & -9 & 5 & 3 \\ 1 & 1 & 3 & -3 & 7 \end{bmatrix}
\]

(a) (4 pts.) Find numbers \( a \) and \( b \) so that \( \text{Col} \ A \) is a subspace of \( \mathbb{R}^a \), and \( \text{Nul} \ A \) and \( \text{Row} \ A \) are subspaces of \( \mathbb{R}^b \).

(b) (6 pts.) Find \( \dim \text{Col} \ A \) and a basis for \( \text{Col} \ A \).

(c) (6 pts.) Find \( \dim \text{Nul} \ A \) and a basis for \( \text{Nul} \ A \).