Answer each of the following questions. Show all work, as partial credit may be given.

1. (5 pts. each) Use substitution to evaluate the following definite integrals.
   
   (a) \[ \int_{0}^{\pi/2} \sin^4(x) \cos(x) \, dx \]
   
   (b) \[ \int_{0}^{1} \theta (1 - \theta)^{1/2} \, d\theta \]

2. (10 pts.) Suppose that the base of a solid lies in the first quadrant and is bounded by the \( x \)-axis, the \( y \)-axis, and the line \( x + y = 3 \). Find the volume of the solid if the cross-sections of the solid perpendicular to the \( x \)-axis are semi-circles.

3. (10 pts. each) Consider the region in the first quadrant bounded by the \( x \)-axis, the curve \( y = 8x^3 \), and the line \( x = 1 \).
   
   (a) Use the method of disks to find the volume of the solid obtained by rotating the above region about the \( x \)-axis.
   
   (b) Using the method of cylindrical shells, find the volume of the solid obtained by rotating the above region about the \( y \)-axis.
   
   (c) Use the method of washers to set up an integral giving the volume of the solid obtained by rotating the above region about the \( y \)-axis. DO NOT EVALUATE.
   
   (d) Use any method you like to set up an integral giving the volume of the solid obtained by rotating the above region about the line \( y = 8 \). DO NOT EVALUATE.

4. (10 pts.) Find the length of the curve given by \( y = x^{1/2} - (1/3)x^{3/2}, \ 1 \leq x \leq 4 \).

5. (10 pts. each) Suppose that a thin plate covers the region in the first quadrant bounded by the \( x \)-axis, the \( y \)-axis and the curve \( y = 1 - x^2 \), and has constant density \( \delta = 1 \).
   
   (a) Find \( M_x \), the moment of the region about the \( x \)-axis.
   
   (b) Find \( M_y \), the moment of the region about the \( y \)-axis.
   
   (c) Find \( M \), the mass of the region, and its center of mass \((\bar{x}, \bar{y})\).