4.3. Differentiation of Logarithmic and Exponential Functions

Derivative of $\ln x$

$$\frac{d}{dx}(\ln x) = \frac{1}{x} \quad \text{for} \quad x > 0$$

Example

Differentiate the function $f(x) = x \ln \sqrt{x}$. 
The Chain Rule for Logarithmic Functions

If $u(x)$ is a differentiable function of $x$, then

$$
\frac{d}{dx} \ln u(x) = \frac{u'(x)}{u(x)}
$$

Example

Differentiate the function $f(x) = \ln(x^2 + 1)$. 
Example
Differentiate the function \( f(x) = \ln(x^3 - 5x + 4) \).
Example
Find an equation for the tangent line to \( y = x + \ln x \) at the point where \( x = e \).
Differentiation of Exponential Functions

The Derivative of the Exponential Function

\[
\frac{d}{dx} (e^x) = e^x \quad \text{for every real number} \quad x
\]

Example

Differentiate the function \( f(x) = \frac{e^x}{x} \).
Differentiation of Exponential Functions

The Chain Rule for Exponential Functions
If \( u(x) \) is a differentiable function of \( x \), then

\[
\frac{d}{dx} e^{u(x)} = e^{u(x)} u'(x)
\]

Example
Differentiate the function \( f(x) = xe^{2x} \).
Differentiation of Exponential Functions

Example

Find the largest and smallest values of the function \( F(x) = e^{x^2 - 2x} \) over the closed interval \( 0 \leq x \leq 2 \).
Logarithmic Differentiation

Differentiating a function that involves products, quotients, or powers can often be simplified by first taking the logarithm of the function.

Step 1. Take logarithms of both sides of the expression for $f(x)$ and simplify the resulting equation.

Step 2. Use the chain rule to differentiate both sides.

Step 3. Multiply both sides with $f(x)$ to get $f'(x)$. 
Example

Use logarithmic differentiation to find the derivative of

\[ f(x) = 4\sqrt[4]{\frac{2x + 1}{1 - 3x}}. \]
Logarithmic Differentiation

Example

Use logarithmic differentiation to find the derivative of

\[ f(x) = \frac{e^{3x}(x^2 + 5)}{(1 - x)^5}. \]