Absolute Maxima and Minima of a function

Let $f$ be a function defined on an interval $I$ containing the number $c$. Then

- $f(c)$ is the **absolute maximum** of $f$ on $I$ if $f(c) \geq f(x)$ for all $x$ in $I$.
- $f(c)$ is the **absolute minimum** of $f$ on $I$ if $f(c) \leq f(x)$ for all $x$ in $I$.

Collectively, absolute maxima and minima are called **absolute extrema**.
Absolute Extrema on a Closed interval

How to Find the Absolute Extrema of a Continuous Function \( f \) on \( a \leq x \leq b \)

Step 1. Find all critical numbers of \( f \) in \( a < x < b \).
Step 2. Compute \( f(x) \) at the critical numbers found in step 1 and at the endpoints \( x = a \) and \( x = b \).
Step 3. The largest and smallest values found in step 2 are, respectively, the absolute maximum and absolute minimum values of \( f(x) \) on \( a \leq x \leq b \).
Example
Find the absolute maximum and absolute minimum (if any) of

$$f(x) = x^3 + 3x^2 + 1; \quad -3 \leq x \leq 2.$$
Absolute Extrema on a Closed interval

Example
Find the absolute maximum and absolute minimum (if any) of

\[ f(t) = \frac{t^2}{t - 1}; \quad -2 \leq t \leq \frac{1}{2}. \]
Absolute Extrema on a general interval

Example
Find the absolute maximum and absolute minimum (if any) of

\[ f(u) = u + \frac{16}{u}; \quad u > 0. \]