

## 2.5 The Banach Space $C[a, b]$

- In this section we pursue a few of the ideas stated on the last slide of section 2.4.
- This means to take some of the ideas we've considered so far for real numbers and try to develop similar ideas in other settings, namely in “**function spaces**”.
- The first thing we developed for real numbers is the idea of a **sequence**, so we consider that first.

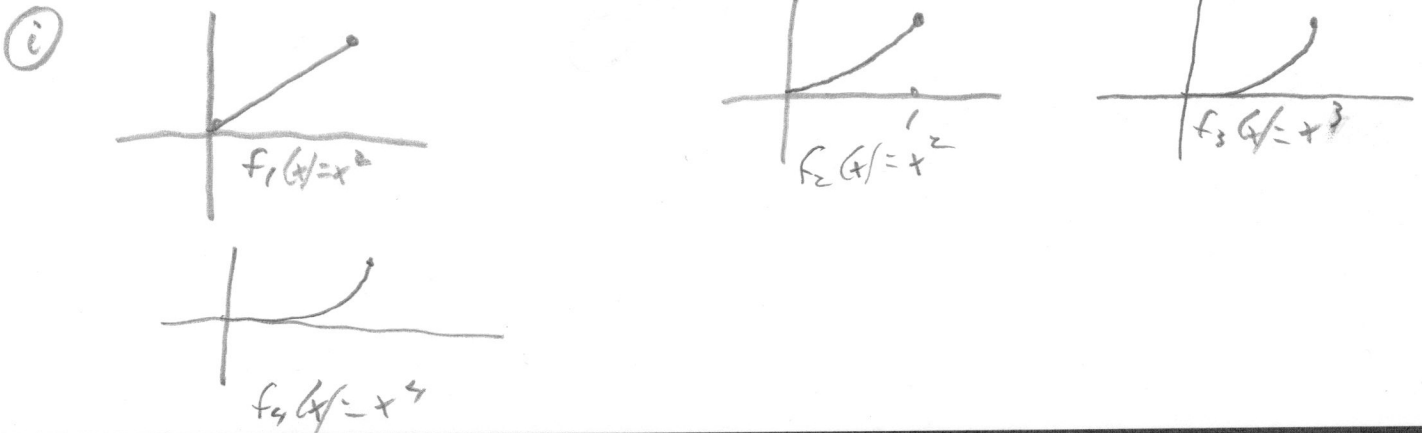
## Sequences of functions

- Let  $D$  be any subset of  $\mathbb{R}$ .
- Suppose for each  $n \in \mathbb{N}$  we have a real-valued function  $f_n$  with  $f_n : D \rightarrow \mathbb{R}$ . (Note: The term "function" will always mean real-valued function.)
- We refer to  $\{f_n\}_{n=1}^{\infty}$  as a sequence of functions on  $D$ .

Exercise.

Consider the sequence of functions  $f_n : [0, 1] \rightarrow \mathbb{R}$ ,  $f_n(x) = x^n$ .

- Sketch the graph of a few terms of the sequence.
- What is the apparent behavior of the sequence as you can see from the graph? Does it appear to go to some specific function?



(ii) Graphs are approaching the graph of  $f(x) = \begin{cases} 0 & \text{if } 0 \leq x < 1 \\ 1 & \text{if } x = 1 \end{cases}$

